Deterministic PDA's

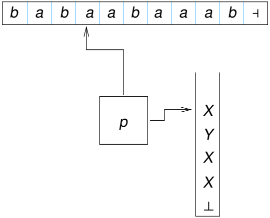
Deepak D'Souza

Department of Computer Science and Automation
Indian Institute of Science, Bangalore.

2 Nov 2021

Outline

Deterministic PDA's



A PDA with restrictions that:

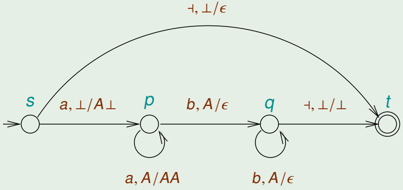
- At most one move possible in any configuration.
  - For any state p, $a \in A$, and $X \in \Gamma$: at most one move of the form $(p, a, X) \rightarrow (q, \gamma)$ or $(p, \epsilon, X) \rightarrow (q, \gamma)$.
  - Effectively, a DPDA must see the current state, and top of stack, and decide whether to make an ϵ-move or read input and move.
- Accepts by final state.
- We need a right-end marker "⊣" for the input.

Example DPDA

Example DPDA for $\{a^n b^n \mid n \geq 0\}$

$$
\begin{array}{lcl}
(s, a, \perp) & \rightarrow & (p, A\perp) \\
(p, a, A) & \rightarrow & (p, AA) \\
(p, b, A) & \rightarrow & (q, \epsilon) \\
(q, b, A) & \rightarrow & (q, \epsilon) \\
(q, \dashv, \perp) & \rightarrow & (t, \perp) \\
(s, \dashv, \perp) & \rightarrow & (t, \perp).
\end{array}
$$

State Diagram of DPDA

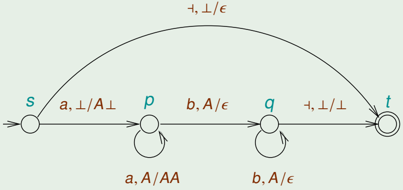


Example DPDA

Example DPDA for $\{a^n b^n \mid n \geq 0\}$

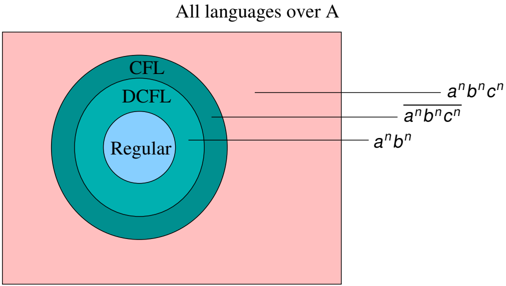
$$
\begin{array}{lcl}
(s, a, \bot) & \rightarrow & (p, A\bot) \\
(p, a, A) & \rightarrow & (p, AA) \\
(p, b, A) & \rightarrow & (q, \epsilon) \\
(q, b, A) & \rightarrow & (q, \epsilon) \\
(q, \dashv, \bot) & \rightarrow & (t, \bot) \\
(s, \dashv, \bot) & \rightarrow & (t, \bot).
\end{array}
$$

State Diagram of DPDA



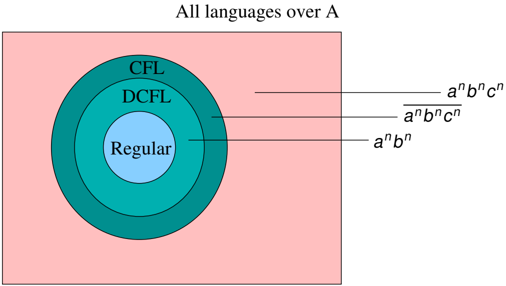
Class of languages accepted by DPDA's are called DCFL's.

Closure Properties of DCFL's



	Closed?
Complementation	

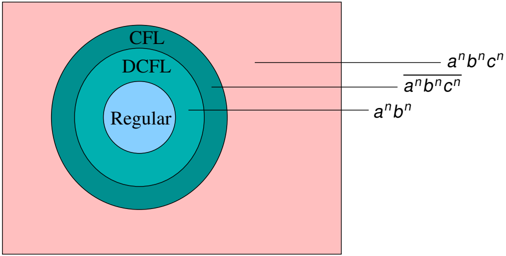
Closure Properties of DCFL's



	Closed?
Complementation	√
Union	

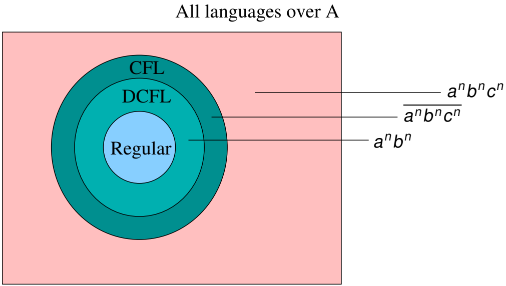
Closure Properties of DCFL's

All languages over A



	Closed?
Complementation	$\surd$
Union	X
Intersection	

Closure Properties of DCFL's



	Closed?
Complementation	√
Union	X
Intersection	X

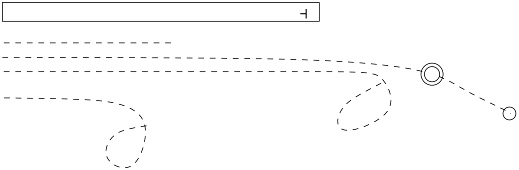
DCFL's are closed under complementation

Theorem (Closure under complementation)

The class of languages definable by Deterministic Pushdown Automata (i.e. DCFL's) is closed under complementation.

Problem with complementing a DPDA

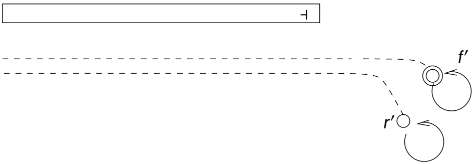
Try flipping final and non-final states.
Problems?



Loops denote an infinite sequence of ϵ-moves.

Desirable form of DPDA

Goal is to convert the DPDA into the form:



That is, always reads its input and reaches a final/reject sink state.

Then we can make r' the unique accepting state, to accept the complement of M.

Construction - Step 1

Let $M = (Q, A, \Gamma, s, \delta, \perp, F)$ be given DPDA. First construct DPDA M' which

- Does not get stuck due to no transition or stack empty.
- Has only "sink" final states.

Construction - Step 1

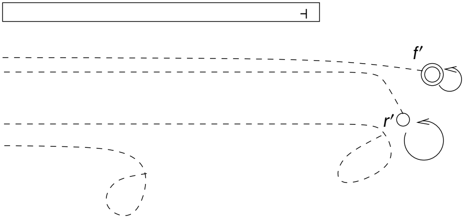
Define $M' = (Q \cup Q' \cup \{s_1, r, r'\}, A, \Gamma \cup \{⫫\}, s_1, \delta', ⫫, F')$ where

- $Q' = \{q' \mid q \in Q\}$ and $F' = \{f' \mid f \in F\}$.
- δ' is obtained from δ as follows:
  - Assume M is "complete" (does not get stuck due to no transition). (If not, add a dead state and add transitions to it.)
  - Make sure M' never empties its stack, keep track of whether we have seen end of input (primed states) or not (unprimed states):

$$
\begin{array}{lcll}
(s_1, \epsilon, ⫫) & \rightarrow & (s, \perp ⫫) & \\
(p, \epsilon, ⫫) & \rightarrow & (r, ⫫) & (p \in Q) \\
(p', \epsilon, ⫫) & \rightarrow & (r', ⫫) & (p' \notin F') \\
(p, ⊣, X) & \rightarrow & (q', \gamma) & \text{if } (p, ⊣, X) \rightarrow (q, \gamma) \in \delta. \\
(p', \epsilon, X) & \rightarrow & (q', \gamma) & \text{if } (p, \epsilon, X) \rightarrow (q, \gamma) \in \delta,\ p \notin F. \\
(r, a, X) & \rightarrow & (r, X) & \\
(r, ⊣, X) & \rightarrow & (r', X) & \\
(r', \epsilon, X) & \rightarrow & (r', X) & \\
(f', \epsilon, X) & \rightarrow & (f', X) & (f \in F)
\end{array}
$$

After Step 1

DPDA M' only has the following kinds of behaviours now:



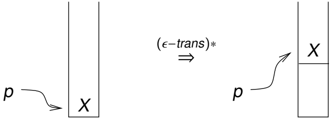
Loops denote an infinite sequence of ϵ-moves.

Construction - Step 2

A spurious transition in M' is a transition of the form $(p, \epsilon, X) \to (q, \gamma)$ such that

$$(p, \epsilon, X) \overset{(\epsilon - trans)*}{\Rightarrow} (p, \epsilon, X\alpha)$$

for some stack contents α.



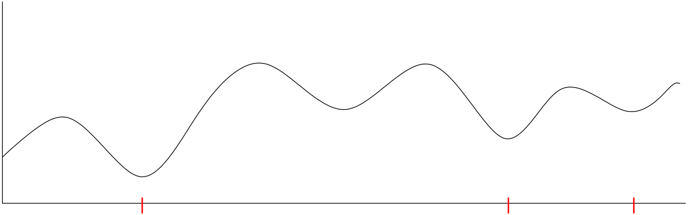
Identify spurious transitions in M' and remove them:
If $(p, \epsilon, X) \to (q, \gamma)$ is a spurious transition and $p \notin F' \cup \{r'\}$, replace it with

$$\begin{array}{lcll} (p, \epsilon, X) & \to & (r, X) & \text{If } p \in Q \\ (p, \epsilon, X) & \to & (r', X) & \text{If } p \in Q' - F'. \end{array}$$

Correctness

Argue that:

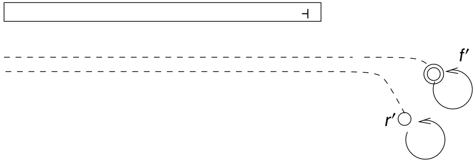
- Deleting a spurious transition (starting from a non-F'-final state) does not change the language of M'.
- All infinite loops use a spurious transition.
  - Look at graph of stack height along an infinite loop, and argue that there are infinitely many future minimas.



  - Further look at transitions applied at these points and observe that one must repeat.
- Thus replacing spurious transitions as described earlier will remove the remaining undesirable loops from M''s behaviours.

Complementing

- Resulting M'' has the desired behaviour (every run either reaches a final sink state or the reject sink state r'.).



- Now make r' unique final state to complement the language of M.

Detecting spurious transitions

Question: How can we effectively detect spurious transitions?

Detecting spurious transitions

Question: How can we effectively detect spurious transitions?

Use algorithm for pushdown reachablity.