Stochastic Systems Lab

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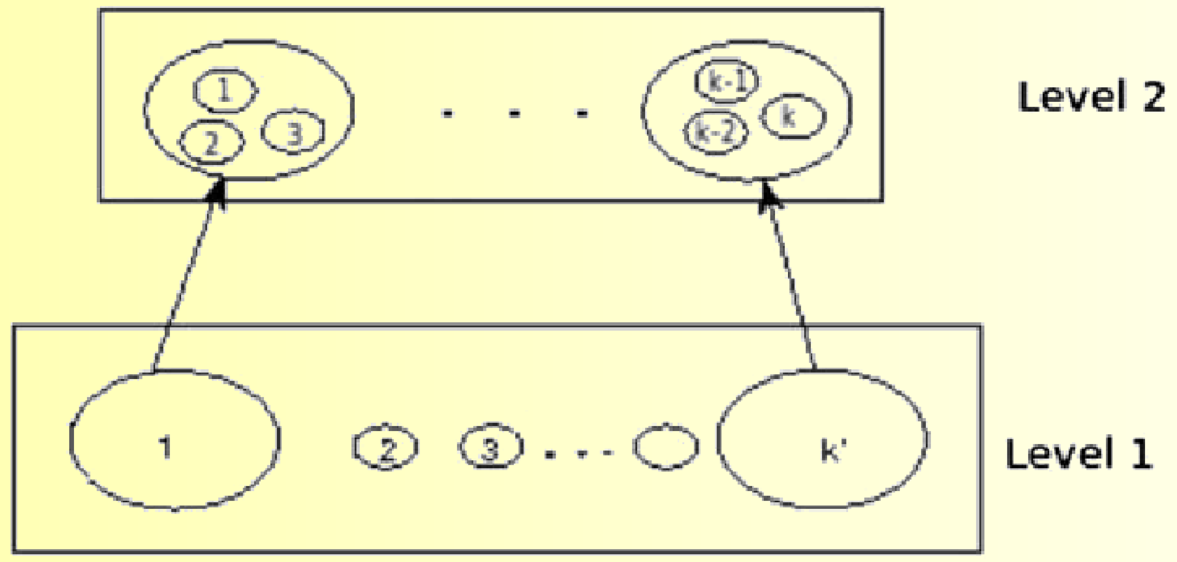
Research Areas

SSL lab conducts research in the area of optimization and control of stochastic dynamic systems -

- Stochastic control and optimization
- Reinforcement learning
- Multi-agent systems and stochastic games
- Machine learning and pattern recognition
- Communication and wireless networks
- Vehicular traffic control
- Data mining

Two-level k-means Clustering



Relation between k and τ:
When $k' = 1$, i.e. we directly cluster dataset into $(R/\tau)^n$ clusters, we can obtain a relationship between the radius threshold τ and the final number of clusters k. The number of distance computations required in this scenario should not be greater than the number of computations required to cluster the dataset into k clusters. Therefore, we have

$$2Nk - k^2 \geq 2N(R/\tau)^n - (R/\tau)^{2n}$$

$$\Rightarrow ((R/\tau)^n - k)((R/\tau)^n + k - 2N) \geq 0$$

Since the maximum values that $(R/\tau)^n$ and k can take are N, the expression $((R/\tau)^n + k - 2N)$ cannot be positive. Hence, the following two inequalities must be satisfied:

$$(R/\tau)^n \leq k \text{ and } (R/\tau)^n \leq (2N - k)$$

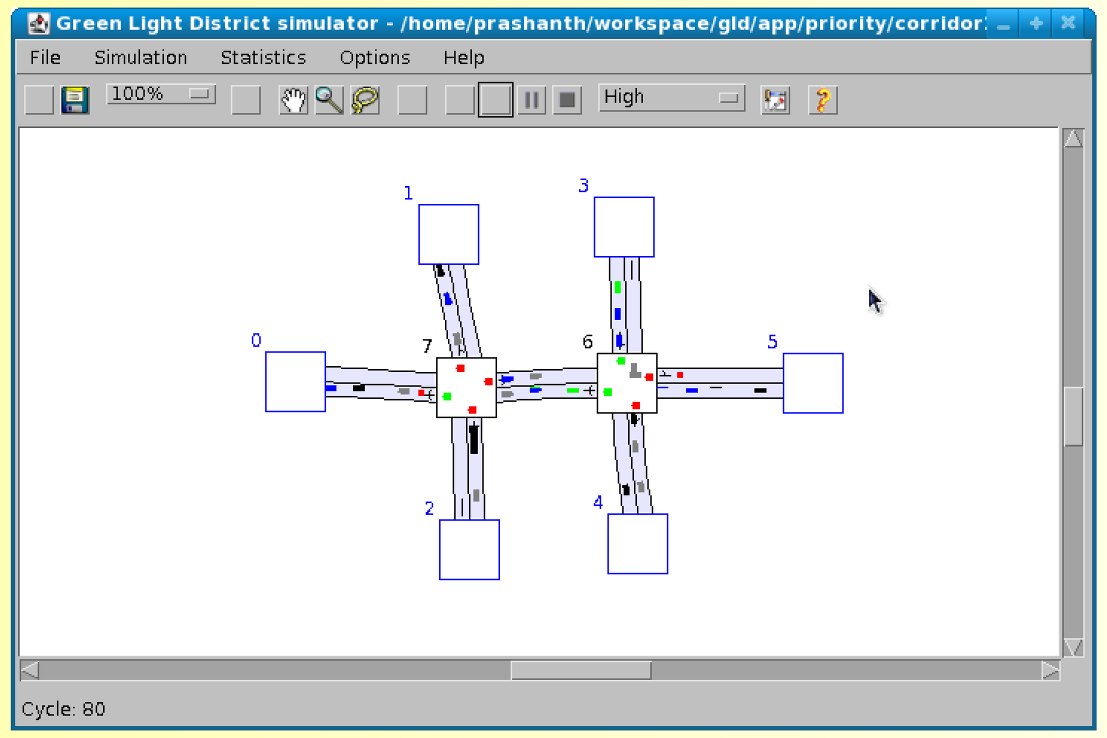
$$\Rightarrow \tau \geq R/(k)^{1/n} \text{ and } \tau \geq R/(2N-k)^{1/n}$$

This gives us the following relation between τ and k:

$$\boxed{max(R/(k)^{1/n}, R/(2N-k)^{1/n}) \leq \tau \leq R}$$

Ref.: Radha Chitta and M. Narasimha Murty. "Two-level k-means clustering algorithm for $k-\tau$ relationship establishment and linear-time classification". Journal of Pattern Recognition, Vol. 43, pp. 796-804, Elsevier, 2010.

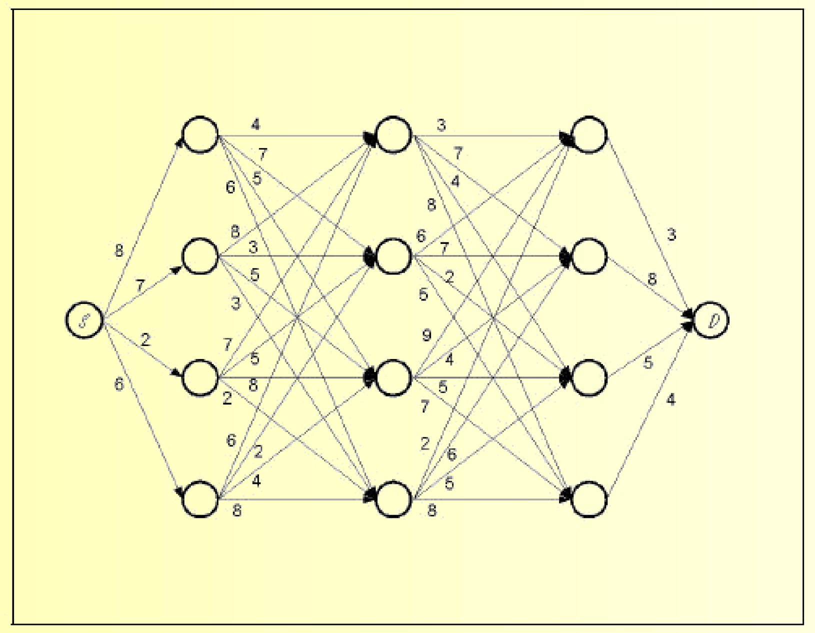
Traffic Signal Control



Traffic signal control via reinforcement learning techniques such as Q-learning.

Ref.: Prashanth L. A. and Shalabh Bhatnagar. "Q-Learning based algorithm for traffic signal control". IEEE Transactions on Vehicular Technology, Submitted 2009.

Ant Colony Optimization



$$T_{ij}^k(t+1) = (1-\rho)T_{ij}^k(t) + \rho Q R_{ij}^k(t)$$

$$X_{ij}^k(t+1) = X_{ij}^k(t) + a(t)X_{ij}^k(t)T_{ij}^k(t+1)$$

Ref.: Sudha Rani K., Lakshmanan K. and S. Bhatnagar. "Ant Colony Optimization Algorithms for Shortest Path Problems". Proceedings of Second Workshop on NET-COOP, LNCS 5425, pp.37-44, Springer, 2008.

Natural Actor-Critic Algorithm

f_{s_t} : state-features
$\psi_{s_t,a_t} = \nabla ln\pi(s_t, a_t)$: state-action features
$\xi_t = c\alpha_t, \beta_t = o(\alpha_t)$
$\sum_t \alpha_t = \sum_t \beta_t = \infty, \sum_t \alpha_t^2, \sum_t \beta_t^2 < \infty$

Avg. Reward Update: $\hat{J}_{t+1} = (1-\xi_t)\hat{J}_t + \xi_t r_{t+1}$
TD Error: $\delta_t = r_{t+1} - \hat{J}_{t+1} + v_t^\top f_{s_{t+1}} - v_t^\top f_{s_t}$
Fisher information matrix inverse:

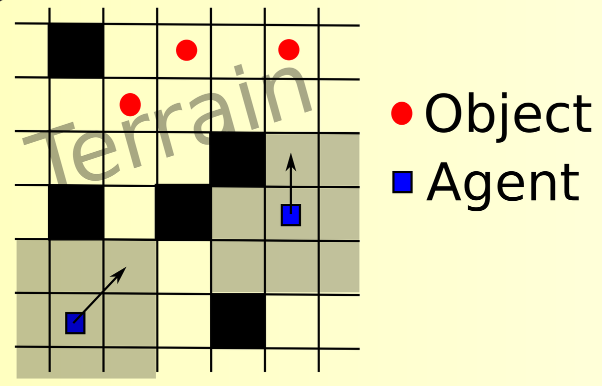
$$G_t^{-1} = \frac{1}{1-\alpha_t}\left[G_{t-1}^{-1} - \alpha_t \frac{(G_{t-1}^{-1}\psi_{s_t a_t})(G_{t-1}^{-1}\psi_{s_t a_t})^\top}{1 - \alpha_t + \alpha_t \psi_{s_t a_t}^\top G_{t-1}^{-1}\psi_{s_t a_t}}\right]$$

Critic Update: $v_{t+1} = v_t + \alpha_t \delta_t f_{s_t}$
Actor Update: $\theta_{t+1} = \Gamma\left(\theta_t + \beta_t G_t^{-1}\delta_t \psi_{s_t a_t}\right)$

Ref.: Shalabh Bhatnagar, Richard S. Sutton, Mohammad Ghavamzadeh and Mark Lee. "Natural Actor-Critic Algorithms". Accepted as a Regular Paper in Automatica, 2009.

Stochastic Games

Stochastic game model of terrain exploration.

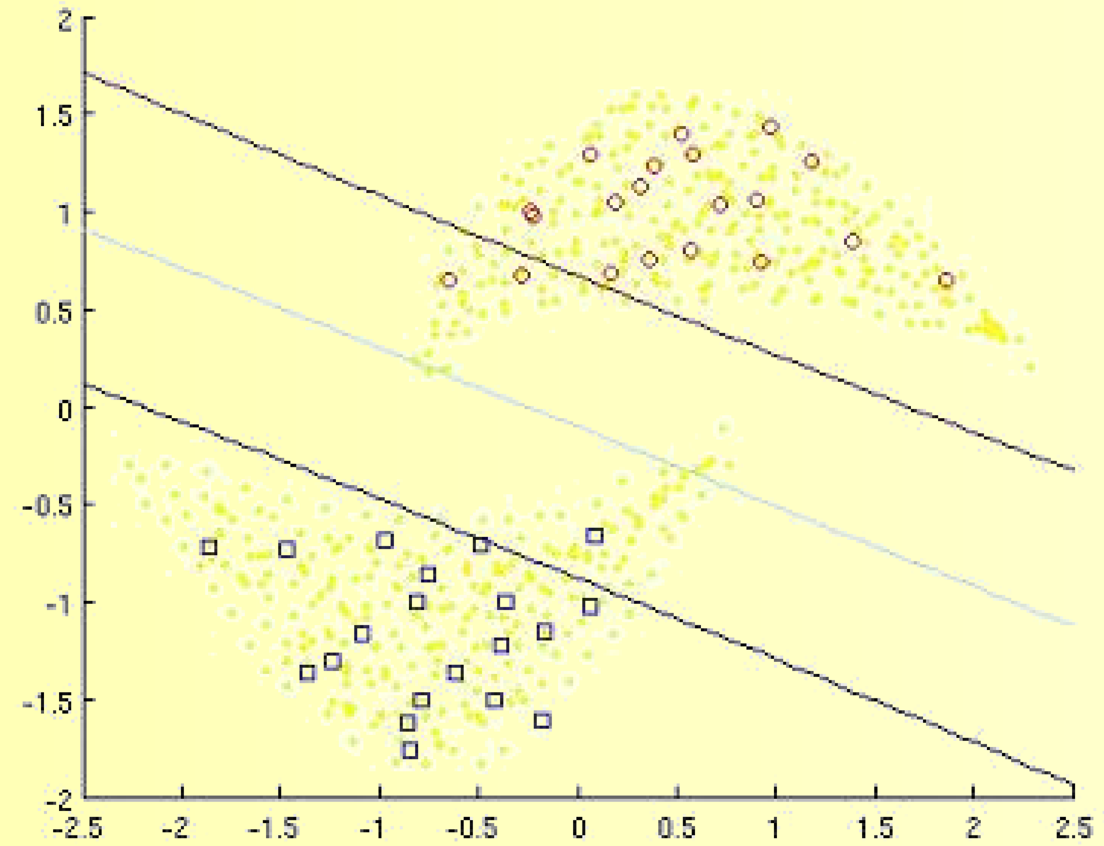


- Any gradient-descent method converges to a Nash equilibrium strategy for discounted reward stochastic games.
- Herskovits two-stage direction method with suitable modifications gives a good offline computational technique.

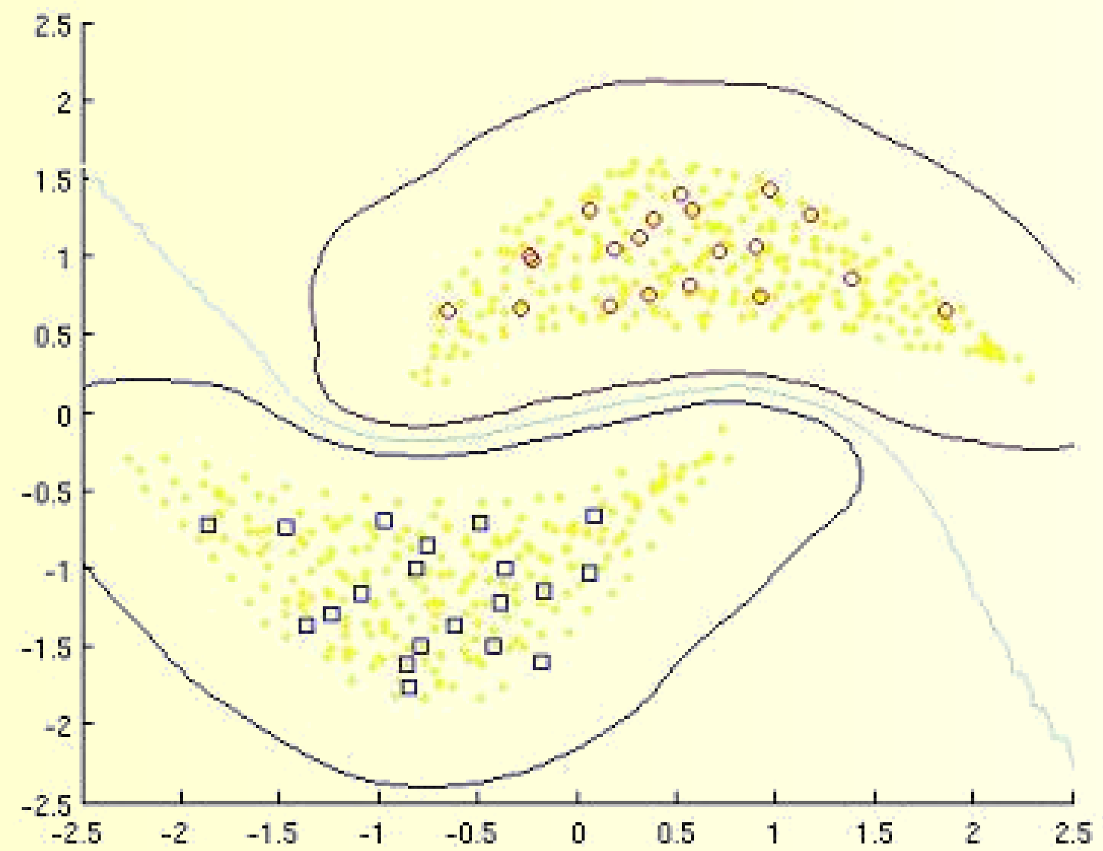
Ref.: Prasad H. L., S. Bhatnagar, and N. Hemachandra. "A computational procedure for general-sum stochastic games". CSA, IISc, Tech. Rep. IISc-CSA-TR-2009-5, May 2009.

Semi-supervised Learning

Gaussian Process Classifier



Semi-supervised Learning GPC



A simple and efficient algorithm for semi-supervised learning using Gaussian processes is designed. Semi-supervised learning uses unlabelled data along with labeled data to improve generalization performance of classifier.

Ref.: Amrish Patel, S. Sundararajan and Shirish Shevade. "Semi-supervised Classification using Sparse Gaussian Process Regression". International Joint conference on Artificial Intelligence (IJCAI), 2009