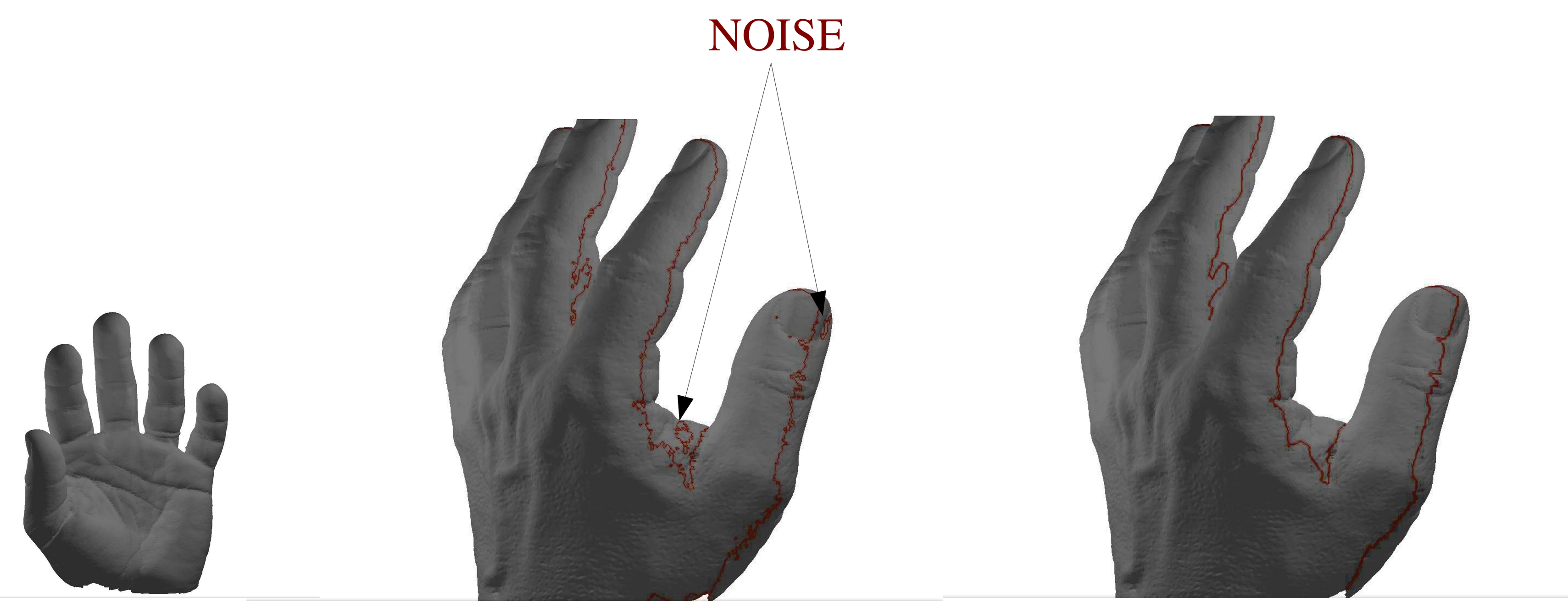
Visualization and Graphics Lab
Indian Institute of Science, Bangalore

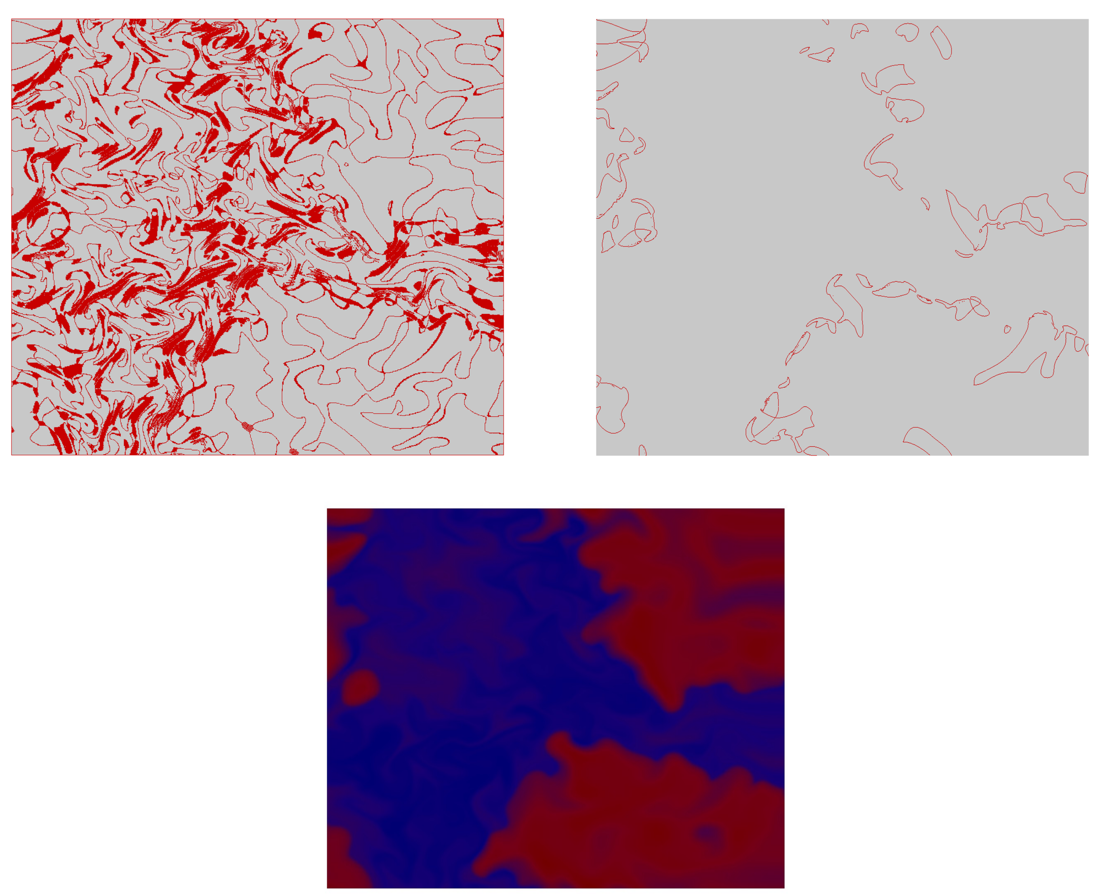
Simplification of Jacobi Sets

The silhouette of the hand model(left) can be represented as the Jacobi set of two functions. The first function is the *x*-coordinate and the second function is the *y*-coordinate. The Jacobi set has noise in the form of small loops(center). Robust computation and simplification gives a clean silhouette(right).



Computation

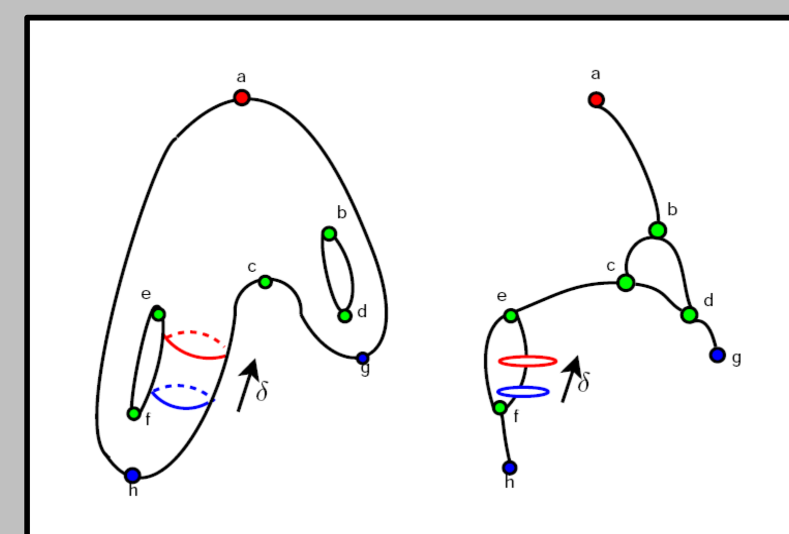
- Comparison measure for two functions *f* and *g* at a point *x* is defined as $\kappa_x = \|\nabla f(x) \times \nabla g(x)\|$.
- Extended comparison measure at *x* is defined as $\kappa_x^S = (\nabla f(x) \times \nabla g(x)) . \hat{n}$, $\hat{n}$ is the unit normal at *x*.
- Jacobi set is computed as the level set of κ_x^S at level zero.

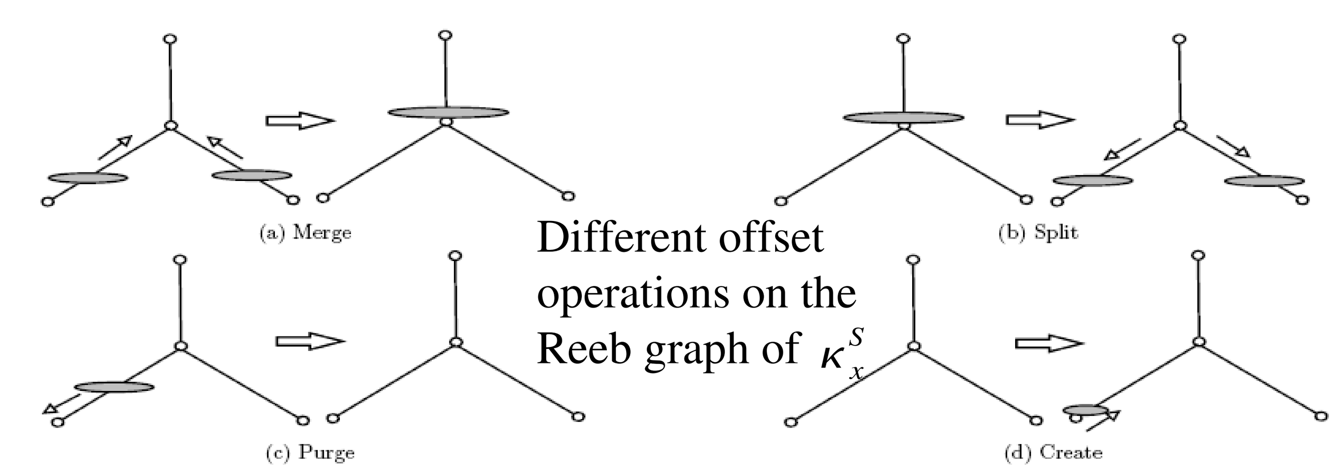


Combustion of Hydrogen
The input data consists of concentrations of hydrogen and oxygen acquired from a combustion simulation. The figure on the top left shows the Jacobi set of the two functions. The simplified Jacobi set (top right) traces the front of the combustion. The figure at the bottom shows the concentration of oxygen using a color map.

Simplification

Offset
Left : Level set components (blue and red) on a manifold. The component shown in blue is offset to the component in red.
Right: Offset shown in the Reeb graph.





Different offset operations on the Reeb graph of κ_x^S

Offset operations used to simplify the Jacobi set. Purge and Merge operations reduce the number of Jacobi set components.

- Hypervolume for a region *R* is defined as

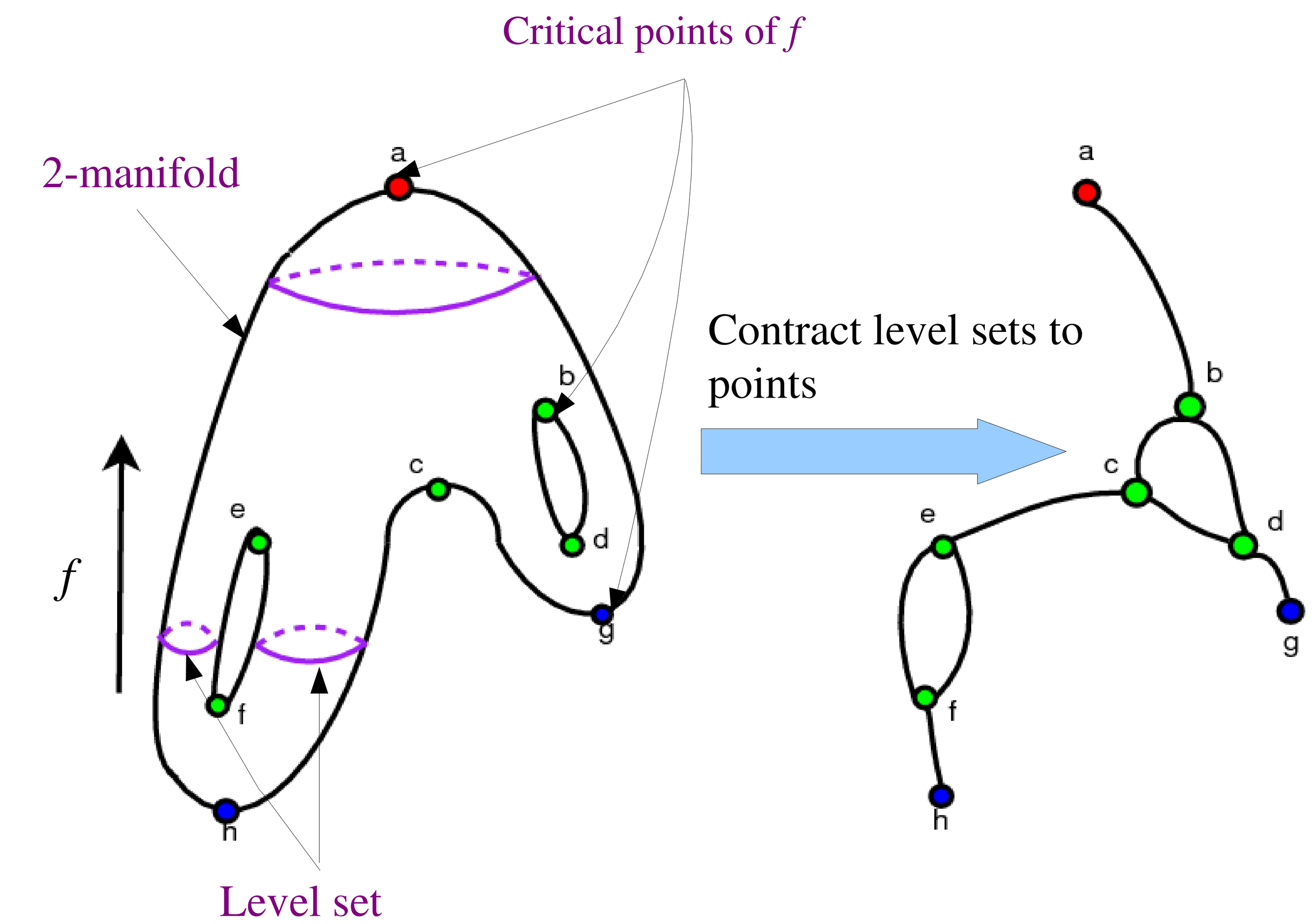
$$H = \frac{1}{Area(M)} \int_R \kappa_x \, dA_x$$

- Error for an offset is computed using the hypervolume for the region swept during the offset

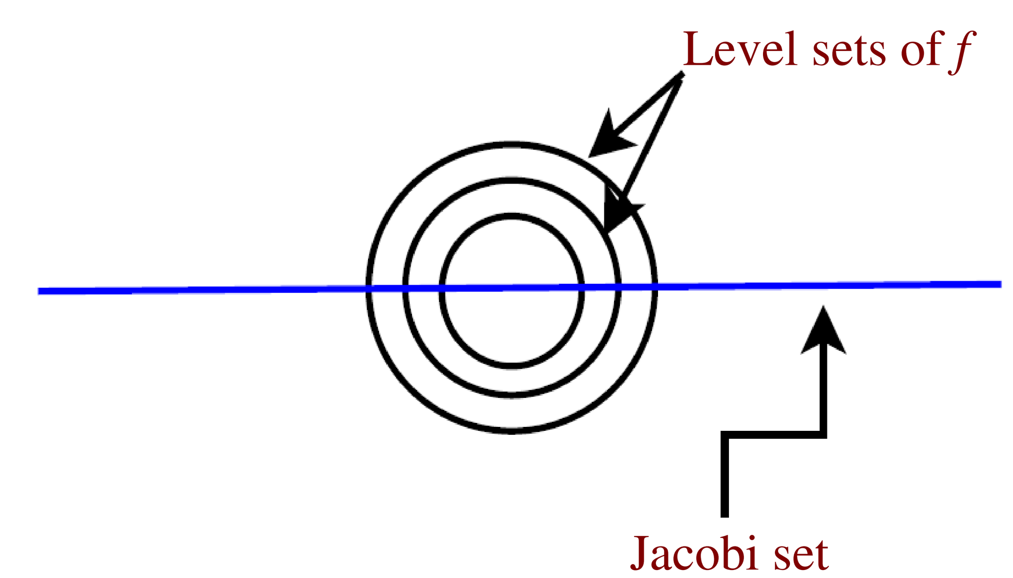
Greedy algorithm to choose offsets so as to minimize total hypervolume

Background

Reeb Graph



Jacobi Set



Jacobi set (blue line) for the functions $f = x^2 + y^2$ and $g = x$. *The Jacobi set* of two functions defined on a 2-manifold is the set of points where the gradients of the functions align with each other.

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