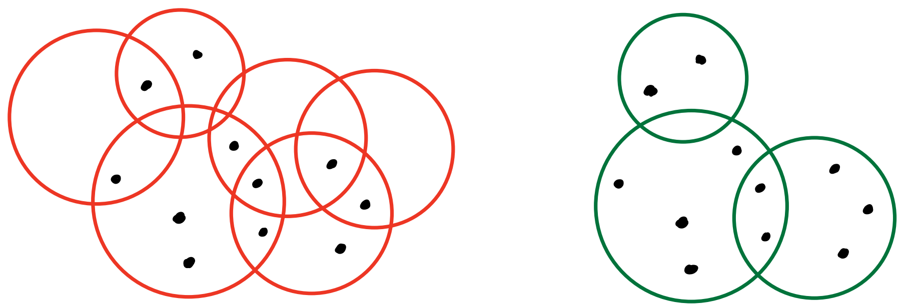
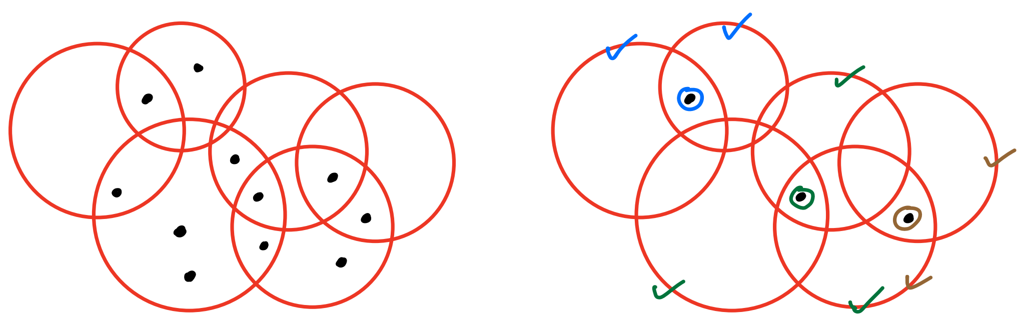
• Geometric Set Cover :

• Geometric set cover (GSC): [Discrete version]
Given m objects I, n points P [weighted/unwt.]
Find min subset $S^* \subseteq I$ that covers all of P.
[Equivalent to hitting set in dual range space]



• Hitting Set (Discrete version):
- Given m objects I, n points P [weighted/unwt.]
Find min subset $S^* \subseteq P$ that stabs all of I.



Now we'll see use of VC-dim & ε-nets to obtain improved approximation.

Ch 20, Notes by Har-Peled. Ch 14 M-U

Sampling: using a small set of observations, estimate properties of an entire sample space.

sample complexity: minimum size sample to obtain the required result.

- Interestingly, one can capture the structure of a distribution / point set by a small subset (ε-net or ε-sample). The size will depend on the complexity of the structure (ranges), but indep of size of point set.

§ VC Dimension: (Vapnik - Chervonenkis dimension).

Definition 14.1: *A range space is a pair* $(X, \mathcal{R})$ *where:*

1. X *is a (finite or infinite) set of* points;
2. $\mathcal{R}$ *is a family of subsets of* X*, called* ranges.

X is also called ground set

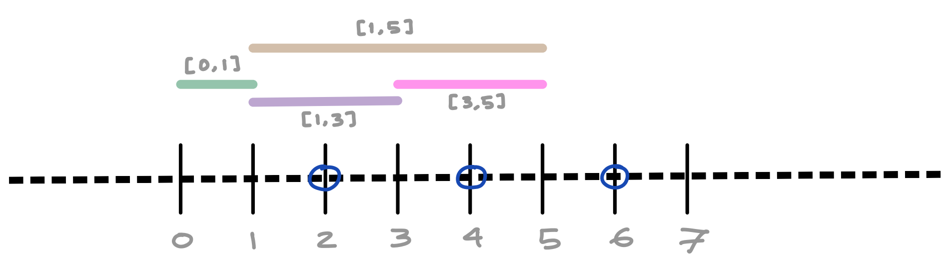
Example of range space:

$X = \mathbb{R}$, $\mathcal{R} = \{ [a,b] \mid [a,b] \subseteq \mathbb{R} \}$.

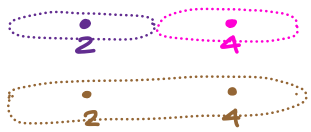
set of all closed intervals

Definition 14.2: *Let* $(X, \mathcal{R})$ *be a range space and let* $S \subseteq X$*. The* projection *of* $\mathcal{R}$ *on* S *is*

$$\mathcal{R}_{|S} = \{R \cap S \mid R \in \mathcal{R}\}.$$



$S = \{2, 4\}$ ϕ



$\mathcal{R}_{|S}$ is the set of all possible subsets of S.

Any two points can be shattered.

$S = \{2, 4, 6\}$ $\mathcal{R}_{|S}$ gives <u>seven</u> of the eight possible subsets of S, except $\{2, 6\}$.

Any 3 points can't be shattered.

— Any interval containing 2 & 6 must contain 4,

Definition 14.3: *Let* $(X, \mathcal{R})$ *be a range space. A set* $S \subseteq X$ *is* shattered *by* $\mathcal{R}$ *if* $|\mathcal{R}_{|S}| = 2^{|S|}$. *The* Vapnik–Chervonenkis (VC) dimension *of a range space* $(X, \mathcal{R})$ *is the maximum cardinality of a set* $S \subseteq X$ *that is shattered by* $\mathcal{R}$. *If there are arbitrarily large finite sets that are shattered by* $\mathcal{R}$, *then the VC dimension is infinite.*

So VC Dim of above range space (with infinite points & intervals) is only 2.

Note: VC dim $(\mathcal{R}) = d$ if there is some set of cardinality d that is shattered by $\mathcal{R}$.
It does not say all sets of cardinality d are shattered by $\mathcal{R}$. To show VC-dim $\leq d$, we need to show all sets of cardinality $> d$ are not shattered by $\mathcal{R}$.

- Sauer-Shelah theorem:

Let $(X, \mathcal{R})$ be a range space with $|X| = n$, VC-dim d. Then, $|\mathcal{R}| \leq n^d$.

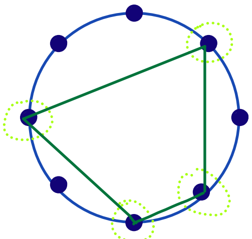
- Low VC-dim intuitively imply cardinality of range space is low.

- **More examples**:

- **Convex sets**: $X = \mathbb{R}^2$, $\mathcal{R} =$ the family of all closed convex sets on the plane.

Claim: This range space has infinite VC-dimension.

→ Need to show, for any $n \in \mathbb{N}$ there exists a set S with $|S| = n$, that can be shattered.



$S_n = \{x_1, \ldots, x_n\}$ be n points on the boundary of a circle.

Any subset $Y \subseteq S_n$, $Y \neq \phi$ defines a convex set that does not contain any points in $S_n \setminus Y$.

Hence, Y is included in the projection of $\mathcal{R}$ on S_n.

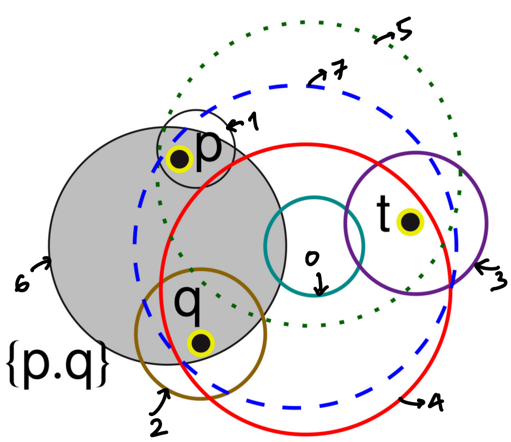
Empty set is also a projection as well.

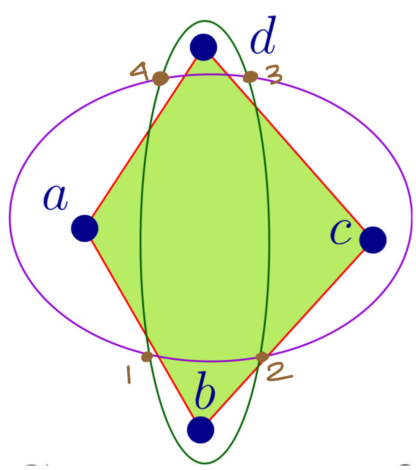
Hence, $\forall n \in \mathbb{N}$, S_n is shattered.

VC-dim 3.

- **Disks**: $X = \mathbb{R}^2$, $\mathcal{R} =$ the family of all disks on the plane.

Observation: For any 3 points on the plane (in general position) one can find eight disks so that the points are shattered.

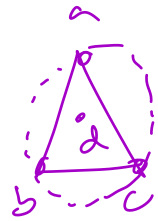




It is ok to show for some set of 3 points. we don't require all set of 3 points

Can disks shatter a set P with four points : $\{a, b, c, d\}$

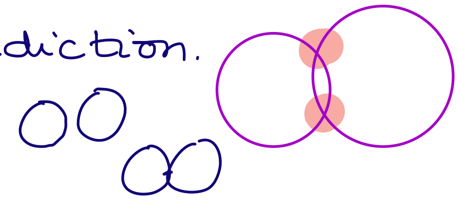
→ Case 1 : convex hull of P has only 3 points on its boundary, say a, b, c.



Then $X = \{a, b, c\}$ can not be obtain as a projection.

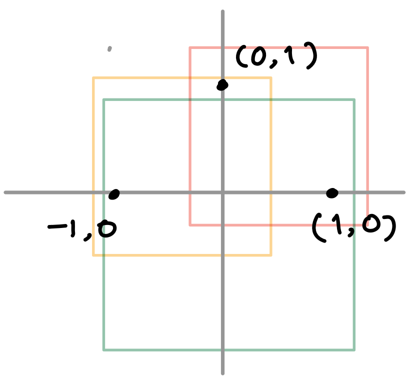
Due to convexity, any disk containing a, b, c must contain d.

→ Case 2 : all 4 points are on the convex hull. Then if we can realize $\{a, c\}$ & $\{b, d\}$ as projections, these two disks will intersect each other at 4 points — a contradiction.



VC-Dim 3

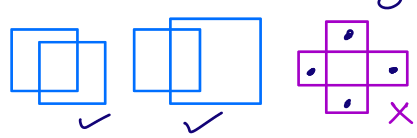
• Squares : $X = \mathbb{R}^2$, $\mathcal{R}$ = the family of all squares on the plane.

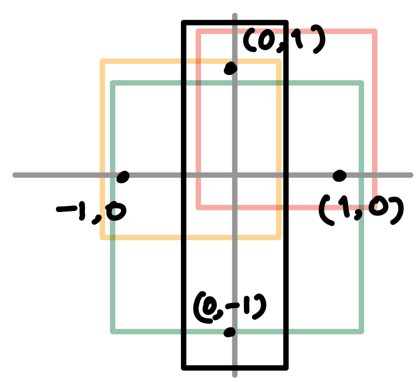


→ A set of 3 points can be shattered.

→ No set of 4 points can be shattered. (similar to above proof for disks)

Two squares can't have crossing intersection.





– Rectangles can support crossing.

• Rectangles : $X = \mathbb{R}^2$, $\mathcal{R}$ = the family of all rectangles on the plane.

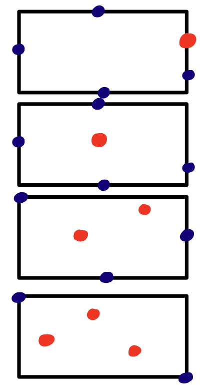
VC-dim 4

No five points can be shattered.

Consider min enclosing rectangle.

① All five points lie on the boundary

② Atleast one point lie inside.



→ We can't shatter the blue points on the boundary.

In general, most simple geometric ranges have low VC-dimension.

§ ε-nets.

- ε-nets are combinatorial object that catches or intersects with every range of sufficient size.

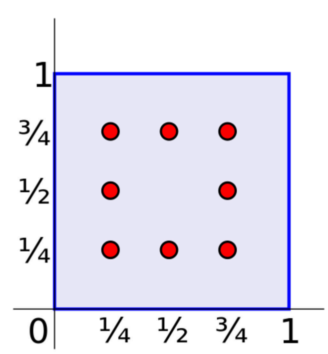
Definition 14.4 [combinatorial definition]: *Let $(X, \mathcal{R})$ be a range space, and let $A \subseteq X$ be a finite subset of X. A set $N \subseteq A$ is a* combinatorial ϵ-net *for A if N has a nonempty intersection with every set $R \in \mathcal{R}$ such that $|R \cap A| \geq \epsilon |A|$.*

Definition 14.5: *Let $(X, \mathcal{R})$ be a range space, and let $\mathcal{D}$ be a probability distribution on X. A set $N \subseteq X$ is an* ϵ-net *for X with respect to $\mathcal{D}$ if for any set $R \in \mathcal{R}$ such that* $\Pr_{\mathcal{D}}(R) \geq \epsilon$, *the set R contains at least one point from N, i.e.,*

$$\forall R \in \mathcal{R}, \;\; \Pr_{\mathcal{D}}(R) \geq \epsilon \Rightarrow R \cap N \neq \emptyset.$$

Here, $\Pr_{\mathcal{D}}(R)$ is the prob. that a point is chosen according to $\mathcal{D}$ is in R.

Note, combinatorial defn. corrs. to the setting when $\mathcal{D}$ is uniform over A.



An ε-net with ε = 1/4 of the unit square in the range space where the ranges are closed filled rectangles.

- **ϵ-net theorem:** Let $(X, \mathcal{R})$ be a range space with VC-dim d and let $\mathcal{D}$ be a prob. distribution on X. For any $0 < \delta, \varepsilon \leq 1/2$, there is an $m = O\left(\frac{d}{\varepsilon} \ln \frac{d}{\varepsilon} + \frac{1}{\varepsilon} \ln \frac{1}{\delta}\right)$ such that a random sample from $\mathcal{D}$ of size m is an ε-net for X with probability at least $1-\delta$.

⊙ $O(d \ln(d\,OPT))$-approximation for hitting set with VC-dimension d.

Hitting set variant:

$n = \#$ elements, $m = \#$ sets.

$X := \{e_1, \ldots, e_n\}$ $\quad$ $R := \{S_1, \ldots, S_m\}$.

Algorithm:

→ Guess OPT (by binary search). $\epsilon = 1/2OPT$.

Initialize:

→ Put $w(e_i) = 1 \;\forall\, i \in [n]$. // start with uniform weights on each element

Loop:

→ Find ϵ-net N_ϵ of size $O\left(\frac{d}{\epsilon} \ln \frac{d}{\epsilon}\right)$.

→ If all sets are hit, return N_ϵ & stop.

→ Else $\exists S_j$ s.t. $S_j \cap N_\epsilon = \phi$ // if some set is not hit by ϵ-net

$w(e_i) = 2w(e_i) \;\forall\, e_i \in S_j$ // Double weights of points in S_j

→ Goto Loop.

→ The algorithm is a variant of multiplicative weight update (MWU). Intuitively, total weight of points increases by a rate $(1+\epsilon)$, and OPT increases by a faster rate $(1+\frac{1}{OPT}) = (1+2\epsilon)$. Thus the algorithm stop quickly w. good guarantee.

• Theorem : If $\exists$ hitting set of size OPT, the doubling process can happen at most $O(OPT \cdot \log \frac{n}{OPT})$ times, and the total weight is at most n^4/OPT^3.

→ Say, H be an optimal set.
For input X, say the set S_j is returned by an iteration.

Then, $w(S_j) \leq \varepsilon \cdot w(X)$

Thus, in each iteration, $w(X)$ becomes at most $w(X) + w(S_j) \leq (1+\varepsilon) w(X)$.

So, total weight of X after k iterations :

$$\boxed{w(X)} \leq n(1+\epsilon)^k \leq \boxed{ne^{\epsilon k}} \qquad [\because 1+\epsilon \leq e^{\epsilon} \; \forall \epsilon > 0.]$$

As H is a hitting set, $H \cap S_j \neq \phi$.
So, at least one element $h \in H$ is doubled in each iteration. Say, h is doubled totally z_h times.

$$w(H) = \sum_{h \in H} 2^{z_h}, \quad \text{where } \sum_{h \in H} z_h \geq k.$$

$$\geq (2^{2\epsilon k}) / 2\epsilon.$$

[Here we have used convexity of exponential function. from Jensen's inequality, $\sum_i p_i \phi(x_i) \geq \phi(\sum p_i x_i)$ where $p_i \geq 0$, $\sum p_i = 1$ & ϕ is convex.

H is optimal i.e. $|H| = 1/2\epsilon$. Take $\phi(x) = 2^x$, $p_i = 2\epsilon \ \forall i \in [|H|]$.

$$\therefore \sum_{h \in H} 2^{z_h} = \frac{1}{2\epsilon}\left[\sum_{h \in H} 2\epsilon \cdot 2^{z_h}\right] \geq \frac{1}{2\epsilon} 2^{\sum 2\epsilon \cdot z_h} \geq \frac{1}{2\epsilon} 2^{2\epsilon \cdot k}. \quad]$$

As $w(H) \leq w(X)$,

$$(2^{2\epsilon k}) / 2\epsilon \leq n e^{\epsilon k} \leq n 2^{\frac{3}{2} \cdot \epsilon k} \quad (\because e \leq 2^{3/2} \approx 2.82)$$

$$\Rightarrow 2^{2\epsilon k - (3\epsilon k/2)} \leq 2n\epsilon$$

$$\Rightarrow 2^{\epsilon k/2} \leq 2n\epsilon \quad \Rightarrow \quad \epsilon k/2 \leq \log(2n\epsilon)$$

$$\Rightarrow k \leq \frac{2}{\epsilon} \log(2n\epsilon) \quad = O(OPT \cdot \log(n/OPT))$$

$$\therefore w(X) \leq n e^{\epsilon k} \leq n e^{2\log(2n\epsilon)} \leq O(n^3/OPT^2). \quad \blacksquare$$

$\Rightarrow$ So we can stop the run if # iterations exceed k.

In special cases, if $\exists$ small ϵ-nets of size $O(d/\epsilon)$, an $O(d)$-approx is obtained.

$\frac{1}{\epsilon} = \Theta(OPT)$.

- Matousek - Siedel - Welzl '90:

For disks in $\mathbb{R}^2$, $\exists$ ϵ-nets of size $O(1/\epsilon)$.

• Aronov - Ezra - Sharir [STOC'09]

Better ε-nets for rectangles : $O(\frac{1}{\varepsilon} \log \log \frac{1}{\varepsilon})$.
used for hitting set.

– also true for dual range spaces of rectangles.
used for set cover.

$\Rightarrow$ as $\varepsilon = \Theta(\frac{1}{OPT}) \Rightarrow$ soln of cost $O(OPT \log\log OPT)$

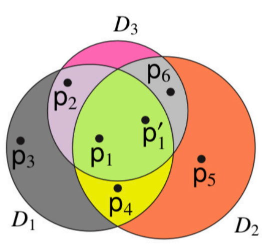
This is still the best-known approx.

Open problem : O(1)-approximation for geometric hitting set/set cover for rectangles.
related by dual range space

Definition 20.2.8. The ***dual range space*** to a range space $\mathsf{S} = (\mathsf{X}, \mathcal{R})$ is the space $\mathsf{S}^\star = (\mathcal{R}, \mathsf{X}^\star)$, where $\mathsf{X}^\star = \{\mathcal{R}_\mathsf{p} \mid \mathsf{p} \in \mathsf{X}\}$.

$S^{**} = S$

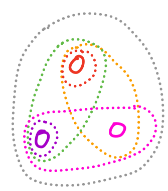
(A)



(B)

	p_1	p_1'	p_2	p_3	p_4	p_5	p_6
D_1	1	1	1	1	1	0	0
D_2	1	1	0	0	1	1	1
D_3	1	1	1	0	0	0	1

	D_1	D_2	D_3
p_1	1	1	1
p_1'	1	1	1
p_2	1	0	1
p_3	1	0	0
p_4	1	1	0
p_5	0	1	0
p_6	0	1	1



(C)

Lemma: Consider a range space $S = (X, \mathcal{R})$ with VC-dim d. Then the dual range space $S^* = (\mathcal{R}, X^*)$ has VC-dim $\leq 2^{d+1}$.

Thus, to solve Geometric Set Cover with Rectangles we solve Geometric hitting set with dual range space of rectangles (which has VC-dim $O(1)$).

• LP-based approach (Even et al.) [Hitting set]

Natural LP: (LP1)

$$\min \sum_{u \in X} x_u = J.$$

$$\text{s.t.} \sum_{u \in S} x_u \geq 1, \quad \forall S \in \mathcal{R}$$

$$x_u \geq 0, \quad \forall u \in X$$

Equivalent LP: (LP2)

$$\max \ \varepsilon$$

$$\text{s.t.} \sum_{i \in S} \mu_i \geq \varepsilon, \ \forall S \in \mathcal{R}$$

$$\sum_{u \in X} \mu_u = 1$$

$$\varepsilon, \mu_u \geq 0, \ \forall u \in X$$

Equivalence proof:

Use substitution $\varepsilon = 1/\sum_{u \in X} x_u$, $\mu_u = \varepsilon \cdot x_u \ \forall u \in X$

$\therefore J^* = 1/\varepsilon^*$.

Algorithm:

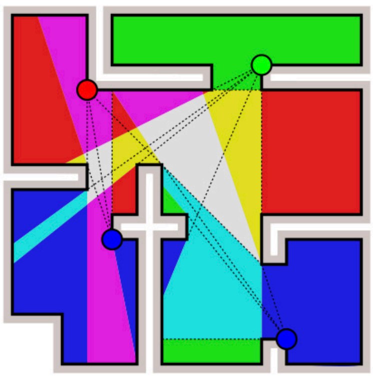
1. Solve LP2 to obtain μ^*, ε^*.
2. Find ε^*-net H with weight$(u) = \mu_u$, $\forall u \in X$.

As $\sum_{i \in S} \mu_i \geq \varepsilon$, $\forall S \in \mathcal{R}$, H is a hitting set.

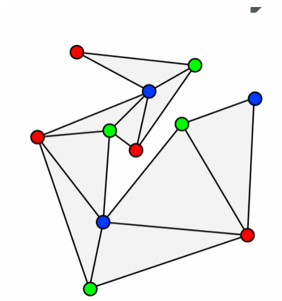
→ Can be extended to weighted setting as well.

• Application: Art Gallery Theorem.

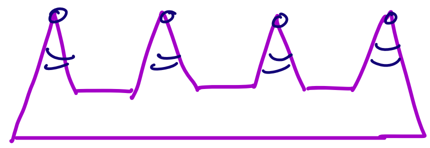
The **art gallery problem** or **museum problem** is a well-studied visibility problem in computational geometry. It originates from a real-world problem of guarding an art gallery with the minimum number of guards who together can observe the whole gallery. In the geometric version of the problem, the layout of the art gallery is represented by a simple polygon and each guard is represented by a point in the polygon. A set S of points is said to guard a polygon if, for every point p in the polygon, there is some $q \in S$ such that the line segment between p and q does not leave the polygon.



Four cameras cover this gallery.

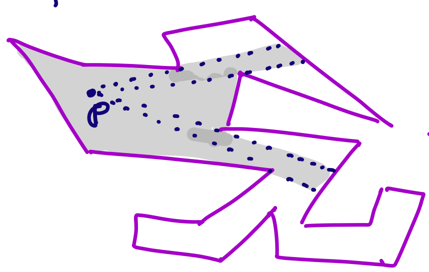


A 3-coloring of the vertices of a triangulated polygon. The blue vertices form a set of three guards, as few as is guaranteed by the art gallery theorem. However, this set is not optimal: the same polygon can be guarded by only two guards.



What about approximation?

- Consider the range space $S = (P, \mathcal{R})$ where $\mathcal{R}$ is the set of all possible visibility polygons inside P.



$V_P(p) = \{q \mid q \in P, \overline{pq} \subseteq P\}$

- Theorem:
  $\text{VC-dim}(S) = O(1)$.
  [Ch 6.4, Har-Peled].

- We want to cover the entire polygon using min # of visibility polygons.

  This is just geometric set cover.

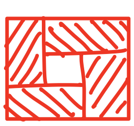
  Using prev algorithms we obtain $O(\log OPT)$-approximation. $\blacksquare$

• Rectangle Packing Problem :

"I think packing problems are appealing to mathematicians and computer scientists because they seem very simple -- just place these items into the container," said researcher and artist Erik Demaine, a professor at the Massachusetts Institute of Technology "Yet they tend to be extremely complicated to actually solve."

• Given n rectangles : (with associated profit)

	Bin packing variant	Knapsack variant
	Pack all rectangles into min # bins. ↕ (nonoverlapping axis-parallel packing) ↔	Pack maximum profit subset of rectangles into a single Knapsack.
Rectangles can be moved ↕ ↔	2D Bin Packing	2D Knapsack
	1.405 [Bansal-K., SODA 14]	1.89 [Galvez et al. '17]
	No APTAS	PTAS might be possible.
d-dim:	$1.69..^{d-1}$ [Caprara '02]	$(1+\epsilon)3^d$ [Sharma] '21
		Open: poly(d)-appx or even f(d) hardness
Guillotine variant	APTAS [Bansal et. al. FOCS '05]	PPTAS [K. et al. SoCG '21]
		PTAS is open



4/3 conjecture : Best 2D BP vs Best Guillotine 2DBP.

Rectangles can only be moved in one direction ↑	uniform Round-SAP	uniform SAP
	$(2+\epsilon)$-appx	1.969
	No APTAS [Kar et al., '22]	[Momke-Wiese '19]
	arbitrary profile: $O(\log\log n)$	$2+\epsilon$ Momke-Wiese '15

Rectangles are fixed	Rectangle Coloring	MWISR
	$O(\log \omega)$	$O(\log\log n)$
	[CW' SODA '21]	

- PTAS for packing squares into knapsack to maximize the packed area.

Given: n squares $I = \{S_1, S_2, \dots, S_n\}$. Square S_i has sidelength s_i.

Goal: Find axis-parallel nonoverlapping packing of max $\underbrace{\text{profit}}_{=\text{area}}$ subset of squares.

① Start with an optimal packing $\mathcal{P}_0$

② Modify $\mathcal{P}_0$ to obtain a structured packing $\mathcal{P}_1$ s.t. $\text{area}(\mathcal{P}_0) \approx \text{area}(\mathcal{P}_1)$.

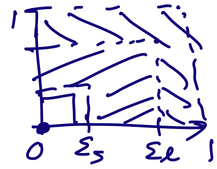
③ Find a packing $\mathcal{P}_2$ in polytime s.t. $\text{area}(\mathcal{P}_1) \approx \text{area}(\mathcal{P}_2)$.

① Item Classification & Shifting:

For two constants ε_{large} & ε_{small} define square S_i small if $s_i \leq \varepsilon_{small}$

large if $s_i \geq \varepsilon_{large}$

& medium if $s_i \in (\varepsilon_{small}, \varepsilon_{large})$.

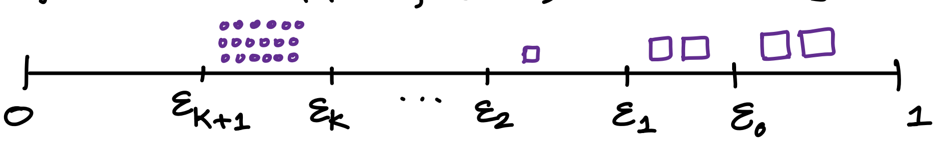


<u>Lemma</u>: For any given $1 > \varepsilon > 0$ and +ve increasing fn $f(\cdot)$, $\exists$ $\varepsilon \geq \varepsilon_{large} > f(\varepsilon_{large}) = \varepsilon_{small} = \Omega_\varepsilon(1)$. s.t. total area of squares with side length in $(\varepsilon_{small}, \varepsilon_{large})$ is at most ε.

$\varepsilon_{small} \geq \underbrace{f^{1/\varepsilon+1}(\varepsilon)}_{g(\varepsilon)}$.

<u>Proof</u>: Take $K = 1/\varepsilon$. $\varepsilon_0 = \varepsilon$.

$\varepsilon_1 = f(\varepsilon)$, $\varepsilon_{i+1} = f(\varepsilon_i)$ $\forall i \in [K]$

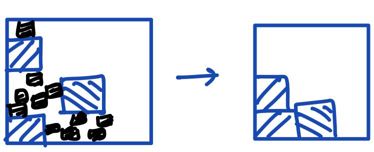


These are $(K+1)$ disjoint ranges: $(\varepsilon_i, \varepsilon_{i-1}]$ $\forall i \in [K+1]$.
So, $\exists$ i s.t. total area of all squares in OPT with sidelength $\in (\varepsilon_i, \varepsilon_{i-1})$ is $\leq 1/(1/\varepsilon) = \varepsilon$.

$\varepsilon_{small} = \varepsilon_i$, $\varepsilon_{large} = \varepsilon_{i-1}$.

So now on we'll ignore medium squares.

- <u>Packing of large squares</u>:

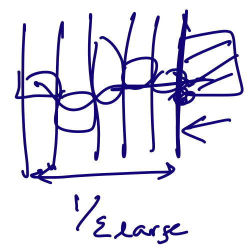


Push all to left & bottom.

\# large items in OPT $\leq \frac{1}{\varepsilon_{large}^2}$.

positions for left bottom corners is $O_{\varepsilon_{large}}(1)$. $\Bigg(\leq \Big(\underbrace{\big(\tfrac{1}{\varepsilon_{large}^2}!\big)}_{\#\text{permutations}} \Big)^{\overbrace{\tfrac{1}{\varepsilon_{large}^2}}^{}} \Bigg)$

#permutations   #items in the chain



② Brute-force for big.

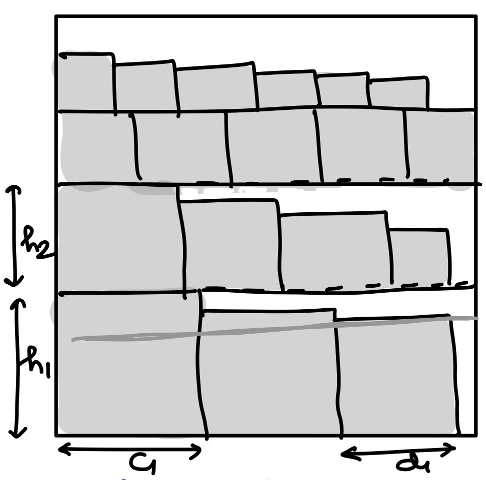
By brute force in $n^{O_{\varepsilon_{large}}(1)}$ time try all possible packings of large squares.

If all items were large, we solve the problem exactly.

From now, assume we "guess" all large squares in OPT. (but their packing can be different from OPT).

• Packing of small squares:

Next-Fit-Decreasing: (NFD / NFDH)



- Sort squares by height
- Squares are packed left-justified on a level until there is insufficient space at the right to accommodate the next rectangle.
- Then start a new level and proceed.

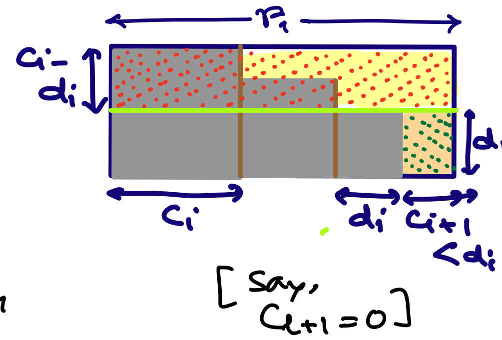
<u>Lemma</u>: Let a set of squares S with sides $\leq \delta$, if NFD cannot place any square in a rectangle $R := r_1 \times r_2$ then total wasted space $\leq \delta(r_1 + r_2)$

$r_1 \geq \delta$, $r_2 \geq \delta$

Proof:

l = # shelves, Length of smallest & largest cube in level i : c_i, d_i.

① Nonincreasing order $\Rightarrow$ $c_{i+1} \le d_i$.



② Total wasted part (red)

$$\le \sum_{i=1}^{l} (c_i - d_i) \cdot r_1$$

$$\le \left[\sum_{i=1}^{l-1} (c_i - c_{i+1}) + c_l - d_l\right] \cdot r_1$$

$$\le (c_1 - d_l) \cdot r_1 \le \delta r_1.$$

Total wasted part (green)

$$\le \sum_{i=1}^{l} d_i^2 \le \delta \sum_{i=1}^{l} d_i \le \delta r_2. \qquad \blacksquare$$

· **Corollary:**

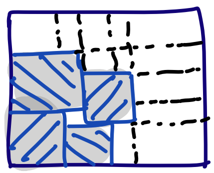
If all squares are small, NFD will pack min{total area of squares, $1 - 2\varepsilon_{small}$}.

$\rightarrow$ So either pack only large or only small $\Rightarrow$ $(2+\epsilon)$-approximation.

• Towards PTAS : Grid-decomposition.

Pack large items in OPT.

Extend their edges to create a grid.



we get $\leq (2 \cdot N_{large} + 2)^2 = \beta \; [O_{\varepsilon_{large}}(1)]$ gridcells.

\# large items.

For a cell $Q := r_1 \times r_2$ if $r_1 \geq \frac{1}{\varepsilon} \varepsilon_{small}$ & $r_2 \geq \frac{1}{\varepsilon} \varepsilon_{small}$, start packing small rectangles in Q by NFD. Else ignore cell Q.

Either we pack all small.

Or, the total wasted space

$$\leq \beta \cdot \underbrace{\frac{1}{\varepsilon} \cdot \varepsilon_{small}}_{\text{ignored cell can have max area}} + \beta \cdot \underbrace{2\,\varepsilon_{small}}_{\text{wasted space in a packed cell}} \leq \beta\left(\frac{1}{\varepsilon} + 2\right) \varepsilon_{small}.$$

We can choose f such that

$$\beta\left(\frac{1}{\varepsilon} + 2\right) \varepsilon_{small} \leq \varepsilon \iff \varepsilon_{small} \leq \frac{\varepsilon^2}{1+2\varepsilon} O_{\varepsilon_{large}}(1).$$

Then we only waste $\leq \varepsilon$ area. $\Rightarrow$ PTAS ∎

- General square packing in 2D-knapsack
  [Jansen-Solis Oba IPCO'08 PTAS]
  [Heydrich-Wiese SODA'17 EPTAS]
  [Jansen et al. PTAS for d-dim cubes into d-dim knapsack, d > 2]

8 Feb 2022

PERFECTLY PACKING A SQUARE BY SQUARES OF NEARLY HARMONIC SIDELENGTH

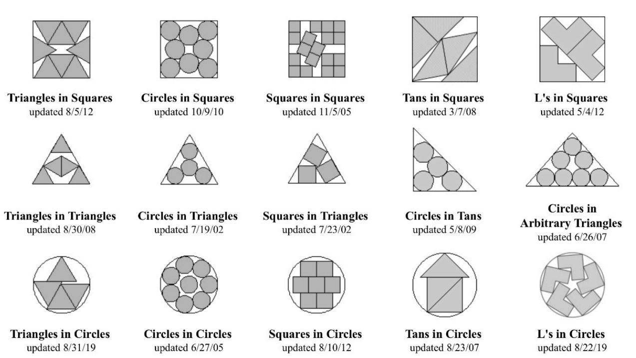
TERENCE TAO

ABSTRACT. A well known open problem of Meir and Moser asks if the squares of sidelength $1/n$ for $n \geq 1$ can be packed perfectly into a square of area $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. In this paper we show that for any $1/2 < t < 1$, and any n_0 that is sufficiently large depending on t, the squares of sidelength n^{-t} for $n \geq n_0$ can be packed perfectly into a square of area $\sum_{n=n_0}^{\infty} \frac{1}{n^{2t}}$. This was previously known for $1/2 < t < 2/3$ (in which case one can take $n_0 = 1$).

erich-friedman.github.io

Erich's Packing Center

Packing Equal Copies



Hoffman's packing puzzle

Article Talk

Hoffman's packing puzzle is an assembly puzzle named after Dean G. Hoffman, who described it in 1978.[1] The puzzle consists of 27 identical rectangular cuboids, each of whose edges have three different lengths. Its goal is to assemble them all to fit within a cube whose edge length is the sum of the three lengths.[2][3]

Hoffman (1981) writes that the first person to solve the puzzle was David A. Klarner, and that typical solution times can range from 20 minutes to multiple hours.[2]

Contents


Hoffman's packing puzzle, disassembled