E0 234 : Homework 3

Deadline : 14th April, 2021, 11 am

Instructions

- Please write your answers using LaTeX. Handwritten answers will not be accepted.

- You are forbidden from consulting the internet. You are strongly encouraged to work on the problems on your own.

- You may discuss these problems with others. However, you must write your own solutions and list your collaborators for each problem. Otherwise, it will be considered as plagiarism.

- Academic dishonesty/plagiarism will be dealt with severe punishment.

- Late submissions are accepted only with prior approval (on or before the day of posting of HW) or medical certificate.

1. *DNF counting.*
   Let F be a DNF formula. Our goal is to estimate the number of satisfying assignments where exactly k variables are set to be true. Give an FPRAS for this problem.

2. *Shuffling cards.*
   Given a deck of n cards, we shuffle by picking a card uniformly at random, and swap it with the topmost card. Show that the mixing time for this process to reach a variation distance of ϵ from the uniform distribution is $O(n^2 \log(1/\epsilon))$.

3. *VC-Dimension.*
   Fine the VC dimensions of the following range spaces.
   (a) $S' := (X, \mathcal{R}')$, where $X = \mathbb{R}$ and $\mathcal{R}'$ is the set of union of k intervals.
   (Hint: for $k = 2$, the corresponding VC-dimension is 4.)
   (b) $S'' := (X, \mathcal{R}'')$, where $X = \mathbb{R}^d$ and $\mathcal{R}''$ is the set of all d-dimensional spheres.

4. *ϵ-net.*
   Let $S := (X, \mathcal{R})$ be a range space with VC-dimension d. Prove that if a random sample M gives an ϵ-net for S with probability at least $1 - \delta$, then $|M|$ is $\Omega(d/\epsilon)$.

Recommended practice problems: (not for submission)

- *Book:* Mitzenmacher-Upfal (2nd edition): 11.2, 11.6, 11.7, 11.8, 11.9, 11.10, 12.3, 12.5, 12.7, 12.8, 12.10, 12.11, 14.1, 14.2, 14.3, 14.4, 14.5, 14.7, 14.10, 14.11, 14.12.