E0 234 : Homework 4

Deadline : 22nd March 2023, 2pm

Instructions

- Please write your answers using LaTeX. Handwritten answers will not be accepted.
- You are forbidden from consulting the internet. You are strongly encouraged to work on the problems on your own.
- You may discuss these problems with another student. However, you must write your own solutions and must mention your collaborator's name. Otherwise, it will be considered as plagiarism.
- Academic dishonesty/plagiarism will be dealt with severe punishment.
- Late submissions are accepted only with prior approval (on or before the day of posting of HW) or a medical certificate.

1. *Tom and Jerry.*
   Tom and Jerry take a random walk on an undirected, connected, non-bipartite graph G. They start at the same time on different vertices, and each moves to a random neighboring vertex at each time step. Tom catches Jerry if they are ever at the same vertex at some time step. Let $n := |V(G)|$ and $m := |E(G)|$. Show that the expected time before Tom catches Jerry is $O(m^2 n)$.
   (*Hint.* Consider a Markov chain whose states are the ordered pair (a, b), where a is the position of Tom and b is the positon of Jerry.)

2. *Lollipop Graph.*
   The *lollipop graph* on n vertices is a clique on $n/2$ vertices connected to a path on $n/2$ vertices.

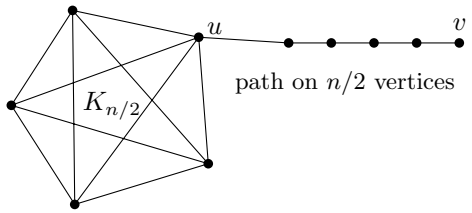

Figure 1: Lollipop Graph

   Let u be the common node between the clique and the path. Let v denote the other end of the path.

   - Show that the expected covering time of a random walk starting at v is $\Theta(n^2)$.
   - Show that the expected covering time of a random walk starting at u is $\Theta(n^3)$.

3. *DNF counting.*
   Let F be a DNF formula. Our goal is to estimate the number of satisfying assignments where exactly k variables are set to be true. Give an FPRAS for this problem.

4. *Shuffling cards.*
   Given a deck of n cards, we shuffle by picking a card uniformly at random, and swapping it with the topmost card. Show that the mixing time for this process to reach a variation distance of ϵ from the uniform distribution is $O(n^2 \log(1/\epsilon))$.

Recommended practice problems (not for grading)

1. *Book:* **Mitzenmacher-Upfal (2nd edition):** 11.5, 11.6, 11.8, 11.9, 11.10, 11.12; 12.2, 12.5, 12.7, 12.8, 12.9, 12.11, 12.12.