OLTSP

TSP on a connected metric space M (offline)
a distinguished point o (origin of M)
a sequence of pairs $x_i\langle t_i, p_i\rangle$ also $0 \le t_i \le t_j$ if $i<j$

Server is at o at time 0, doesn't move faster than unit speed.

Consistency : α-consistent if $c(0) = \alpha$

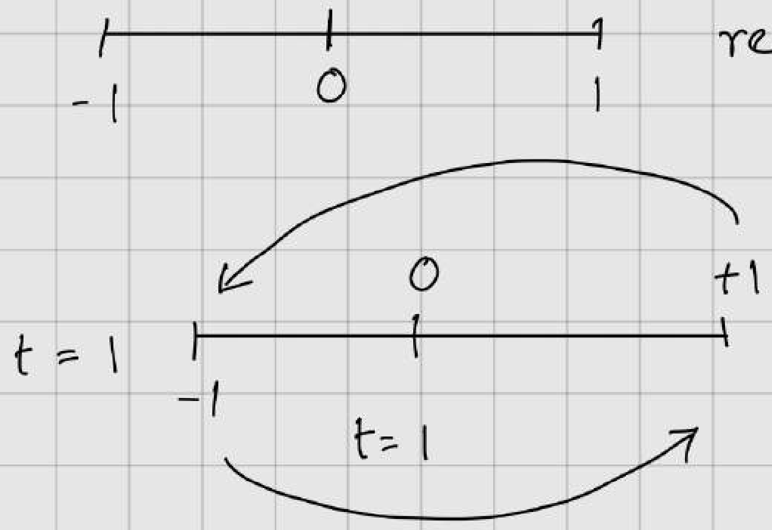
Robustness : β-robust if $c(\varepsilon) \le \beta$ $\forall \varepsilon$

Smoothness : α-consistent, $f(\varepsilon)$-smooth if
$Z^{ALG} \le \alpha \cdot Z^{OPT} + f(\varepsilon)$ for some cts
f^n s.t. $f(0) = 0$

H-OLTSP (Homing eventually returns to origin)
lower bound of 2, 2 comp. algo.
STATE of the art : polytime, 2·65 comp

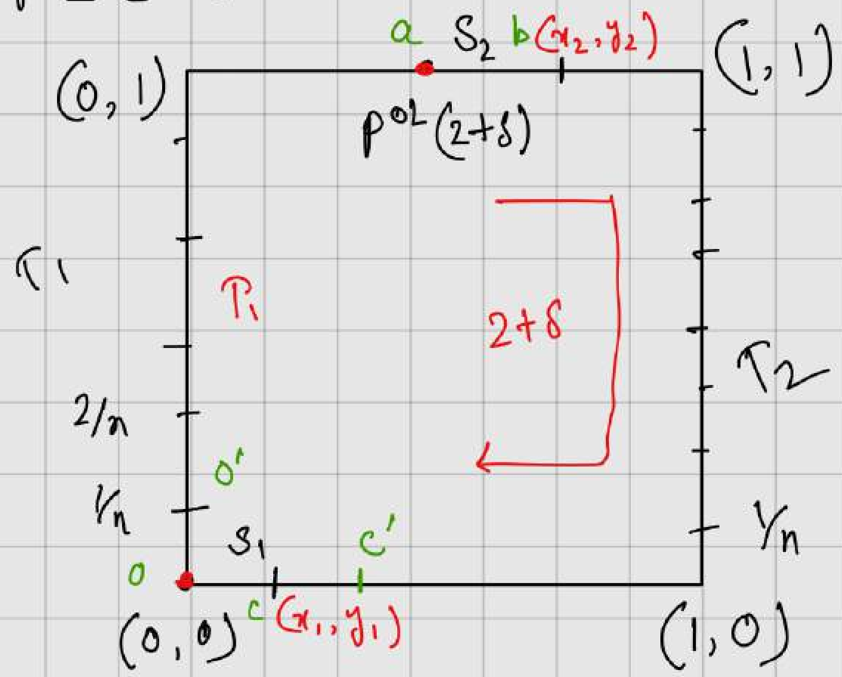
i) NID : a sequence with arbitary size over requests
ii) ID : a " " same no. of request as actual input
iii) prediction of last arrival time

at time t, $p(t)$ is pos^n of the server and U_t is the set of presented unserviced requests at time t.



request at +1 or −1
at time $t=1$ depending
on server is at −1 or +1
adversary done in 1
(just be there)
algo takes at least 2
(2 if at 0 when $t=1$)

Thm 3.2 : For any $\varepsilon > 0$, any ρ-competitive algo for H-OLTSP for metric spaces in $\mathcal{M}$ has $\rho \geq 2-\varepsilon$



t=

For some δ $(0 \leq \delta \leq 2)$, at $t = 2+\delta$, the server must be in one of the two points at dist. $2-\delta$ from origin.

$f : [0,2] \to [0,2]$: distance from $(1,1)$ at time $2+x$

$$g(x) = f(x) - x$$

$g(0) \geq 0 \quad g(2) \leq 0$

g is cts, by IVT for some $0 \leq \delta \leq 2$, $g(\delta) = 0$

$\Rightarrow f(\delta) = \delta$

$|S_1| + |S_2| \leq \delta$ as server is at $2-\delta$ distance from origin and travelled them twice

$|T_1| + |S_1| + |S_2| \leq 2$

So, no requested point of $T_2 = [(x_2, y_2), (x_1, y_1)]$ has been touched. $(|T_2| \geq 2)$

At time $(2+\delta)$, request $\forall$ points on $T_1 = [(0,0), p^{OL}(2+\delta)]$ of length $2-\delta$ is given.

OPT: Anti-clw tour of the square as no point visited before its request time. as it will move over $|S_1| + |T_2| + |S_2| = 4 - (2-\delta) = 2+\delta$

Online: (i) $Z^{OL} \geq (2+\delta) + 2\left(2 - \frac{1}{n}\right) + (2-\delta)$ $\quad a \to c' \to a \to 0$

$= 8 - 2/n$

(ii) $Z^{OL} = (2+\delta) + 2\left(2 - \delta - \frac{1}{n}\right) + (2+\delta)$ $\quad a \to 0' \to a \to 0$

Ratio: $\dfrac{8 - 2/n}{4} = 2 - \dfrac{1}{2n}$

Any ρ-competitive algo for H-OLTSP on $\mathbb{R}$ has $\rho \geq (9+\sqrt{17})/8 \approx 1.64$

$\longrightarrow$ Say OL is a ρ-comp algo with $\rho > (9+\sqrt{17})/8$

$$2\rho - 3 = \frac{9+\sqrt{17}}{4} - 3 = \frac{\sqrt{17}-3}{4} < 1$$

if. $p^{OL}(1) > 2\rho - 3$

Request at point -1, $Z^{OL} > 1 + (2\rho - 3) + 2 = 2\rho$

$$Z^* = 2$$

So, $Z^{OL}/Z^* > \rho$

So, algo won't be ρ competitive.

Hence, $p^{OL}(1) \in [-(2\rho-3), (2\rho-3)]$

$t=1$, issue requests at $t=1$ and $t=-1$

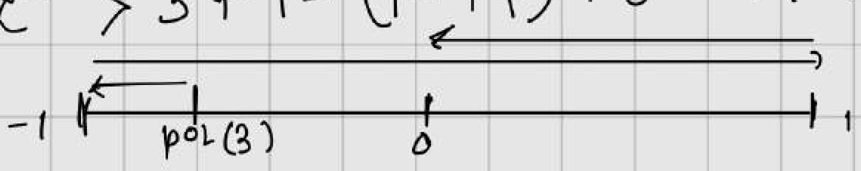
By $t=3$, both requests won't be completed (WLOG. Say request at -1 was not served)

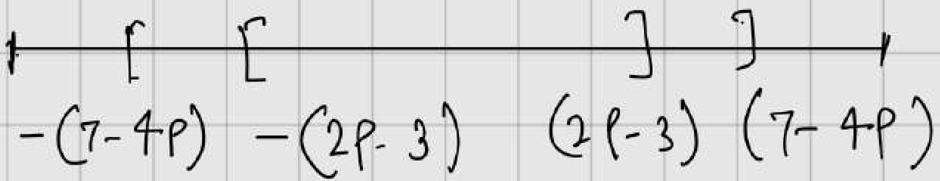
Claim: If $-(7-4\rho) < p^{OL}(3) < (7-4\rho)$, Then OL can't be ρ-comp.

$t=3$, issue requests at $t=1$ (if $p^*(3) = 1$)

Then, $Z^* = 4$

But, $Z^{OL} > 3 + 1 - (7-4\rho) + 3 = 4\rho \Rightarrow \frac{Z^{OL}}{Z^*} > \rho$





1. at $t = 3$, $+1$ hasn't been served and $-1 \leq p^{OL}(3) \leq -(7-4\rho)$
or $(7-4\rho) \leq p^{OL}(3) \leq 1$

2. $+1$ has been served and $(7-4\rho) \leq p^{OL}(3) \leq 1$

$$1 + (1 - (2\rho - 3)) + (1 + (2\rho - 3)) = 3$$

So suppose server is near 1 (within $1 - (7-4\rho)$) and not served the extreme on -1.

algo passes 0 at $t < 4\rho - 2$

$$3 + q \leq 4\rho - 2 \Rightarrow \boxed{q \leq 4\rho - 5}$$

at $(3+q)$, adv. can be at $(1+q)$, place a request there and go to 0.

$$z^* = (3+q) + (1+q) = 4 + 2q$$

$$z^{OL} = (3+q) + 2 + 2(1+q) = 7 + 3q$$

So, $\rho \geq \dfrac{7+3q}{4+2q}$ (↓ing f^n of q)

So, $\rho \geq \dfrac{7+3(4\rho-5)}{4+2(4\rho-5)}$

$\Rightarrow \rho(8\rho - 6) \geq 12\rho - 8$

$\Rightarrow 8\rho^2 - 18\rho + 8 \geq 0$

$\Rightarrow 4\rho^2 - 9\rho + 4 \geq 0$

$\Rightarrow \rho \geq \dfrac{9 + \sqrt{17}}{8}$

GTR (Greedily Travelling b/w Requests)

At time t, when a new request is presented the online server's pos., $p^{GTR}(t)$ is on the shortest path b/w x and y in S (all requests and o)

The algo computes and follows shortest route that first visits x or y and then rest of unserved requests.

\# Thm 4.1: GTR is a $5/2$-comp. algo for N-OLTSP and a tight ratio. (for real line)

- Last request is at time t

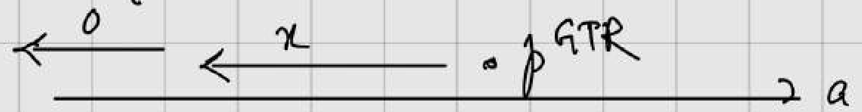
$$z^* \geq t$$

τ is the optimal ham-path on S, constrained to have o as an extreme. (a is the other extreme)

$$z^* \geq |\tau|$$

So, we'll show $z^{GTR} \leq t + \frac{3}{2}|\tau|$

$$\min\left\{d(p^{GTR}(t), x) + d(x, o),\ d(p^{GTR}(t), y) + d(y, a)\right\} \leq \frac{1}{2}|\tau|$$

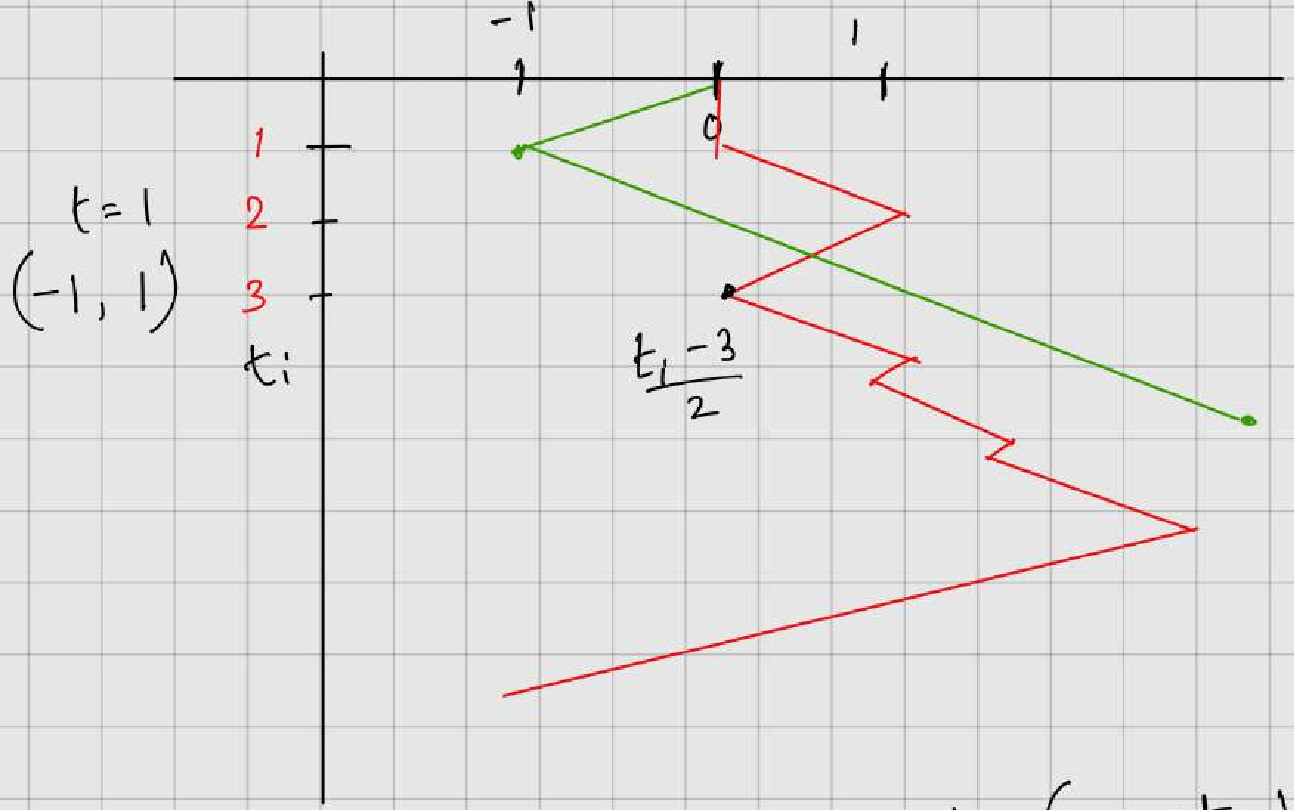


length $\leq \frac{3}{2}|\tau|$

$$z^{OL} \leq t + \frac{3}{2}|\tau| \leq z^* + \frac{3}{2}z^* = \frac{5}{2}z^*$$

GTR

For real line: GTR asks to go to the nearest extreme of the smallest interval containing the requests yet to be visite



Start at time 1 with 2 requests (one at 1, one at -1)

Say, GTR goes to 1 and back to 0 at $t = 3$

$t_0 = 3 \quad p_0 = 1$

$t_i = \frac{5}{3} t_{i-1} - \frac{2}{3} \quad p_i = t_i - 2 \quad i = 1, 2, \ldots, n$ } new set of requests

Now, $Z^* = t_n$ (Reaches p_i at $t_i \ \forall \ i$)

At time t_n, server is in the middle of $[-1, p_n]$

at $i = 0$, $p^{GTR}(3) = 0$

by induction at t_i it leaves $\frac{(t_i - 3)}{2}$ $(\longrightarrow)$

arrival: $t_i + (t_i - 2) - \left(\frac{t_i - 3}{2}\right) = \frac{3t_i}{2} - \frac{1}{2}$

leaves towards (-1) $(\longleftarrow)$

$$(t_i - 2) - \left(\left(\frac{5}{3}t_i - \frac{2}{3}\right) - \left(\frac{3}{2}t_i - \frac{1}{2}\right)\right)$$

$$= \left(t_i - \frac{5}{3}t_i + \frac{3}{2}t_i\right) + \left(-2 + \frac{2}{3} - \frac{1}{2}\right)$$

$$= \frac{5}{6}t_i - \frac{22}{6}$$

$$= \frac{t_{i+1} - 3}{2}$$

t_n:

GTR $\begin{cases} z^{GTR} = t_n + \frac{3}{2}(p_n + 1) = t_n + \frac{3}{2}(t_n - 1) \\ \text{So, ratio} \to \frac{5}{2} \text{ as } n \to \infty \end{cases}$

PAH (Plan-at-home)

1. Whenever server at origin, follow an optimal route that serves all the requests yet to be served and go back to origin.

2. If at t, a new request comes at point x, it might do 2 things

(2a) If $d(x,0) > d(p,0)$, server goes back to the origin (following shortest path from p)

(2b) If $d(x,0) \leq d(p,0)$ then server ignores it until it arrives at origin.

PAH is 2-competitive and it is tight (for real line)

Say, the last request is (t, x)

also, T^* is the optimal tour that starts at the origin, serves all requests and end at origin

So, $Z^* \geq t$ and $Z^* \geq |T^*|$

i) if server was at origin, $Z^{PAH} \leq t + |T^*| \leq 2Z^*$

ii) PAH is not at origin and travelling in some route

if $d(0,x) > d(0,p)$, server goes back to 0.

$$Z^{PAH} = t + d(0,p) + |T^*|$$
$$< t + d(0,x) + |T^*|$$

But, $z^* > t + d(0, x)$ (Opt is at x at time t it has to go back to 0

$\Rightarrow z^{PAH} < 2z^*$

if $d(0,x) \leq d(0,p)$

Say, PAH is following route R (last computed at o), Q set of requests ignored. q is the 1st request served by the adversary. q was presented at time t_q.

P^*_Q is the shortest path starting at q, servicing Q, ending at o. So, $z^* \geq t_q + |P^*_Q|$

Now, $d(0, p(t_q)) \geq d(0, q)$. Server has to travel at most $|R| - d(0,q)$ dist. before coming to 0.

So, arrives before $t_q + |R| - d(0,q)$

$$
\begin{aligned}
z^{PAH} &\leq t_q + |R| - d(0,q) + |T^*_Q| \\
&\leq t_q + |R| - d(0,q) + d(0,q) + |P^*_Q| \\
&= t_q + |P^*_Q| + |R| \\
&\leq z^* + z^* = 2z^*
\end{aligned}
$$

(as T^*_Q is optimal $0 \to 0$ route)

(R is on a subset of requests, so $|R| \leq |P^*|$

Corollary 6: Algo asks server to wait at origin o for time t before following PAH where $t \leq t_n$. It is 2-comp (with access to offline opt route to serve a set of requests)

3-comp using Christofides algo.

5.3 Algo using Christofide's heuristics is 3-comp for H-OLTSP

→ S is the set of requests and o, τ is opt. tour over S

t_n is the time last request was presented

$p^{OL}(t_n)$ is pos. of the server travelling from x to y in S.

$$t \leq Z^*, \quad |\tau| \leq Z^*$$

$$D = d(o,x) + d(x,y) + d(o,y)$$

Now, $Z^* \geq D$ as o, x, y occur in some order in opt. tour

now, $p^{OL}(t) = \min\{d(o,x) + d(x, p^{OL}(t)), d(o,y) + d(y, p^{OL}(t))\}$

$$\leq \frac{D}{2}$$

$$\leq \frac{|Z^*|}{2}$$

$$Z^{OL} \leq t + d(o, p^{OL}(t)) + CHR(S)$$

$$\leq Z^* + \frac{1}{2}Z^* + \frac{3}{2}|\tau|$$

$$\leq 3Z^*$$

No proof of tightness.

LAR-NID (High confidence $\iff$ Low λ)

Input: Current time t

Seq. prediction $\hat{x}$

Confidence $\lambda \in (0,1]$

Current released unserviced req. U_t

- Compute $\hat{T}$ to serve $\hat{x}$ and return to O

When $t < \lambda|\hat{T}|$

If server is at origin $(p(t) = 0)$

Compute T_{U_t} (to serve all unserved requests in U_t and return to 0)

$t + |T_{U_t}| > \lambda|\hat{T}|$

- Find $t_{back} + d(p(t_{back}), 0) = \lambda|\hat{T}|$
- Redesign T'_{U_t} by asking server to go to 0 at t_{back} along shortest path
- Start following T'_{U_t}

Follow T_{U_t}

The server is moving along $T_{U_{t'}}$ for some $t' < t$

- If a new request $x_i = (r_i, p_i)$ comes

Apply PAH

- if $d(p_i, 0) > d(p(t), 0)$ go back to O
- else follow $T_{U_{t'}}$

While $t \geq \lambda|\hat{T}|$

- Wait until $U_t \neq \emptyset$ (Some request arrives)
- Follow $\hat{T}$ until server is back at 0
- Follow PAH(t, U_t) (Serve the remaining requests)

- It is $(1.5+\lambda)$ - consistent. $\lambda \in (0,1]$

→ Say prediction is perfect, $x = \hat{x}$ and $|\hat{T}| = Z^{OPT}$

Server follows PAH till $\lambda|\hat{T}|$ and follows predicted route if $U_{\lambda|\hat{T}|} \neq \emptyset$, o/w waits

if $U_{\lambda|\hat{T}|} \neq \emptyset$, $Z^{ALG} = \lambda|\hat{T}| + |\hat{T}| \leq (1+\lambda) Z^{OPT}$

if stops, Say waits till t_j $\left(t_j = \min_t \{t > \lambda|\hat{T}| : U_t \neq \emptyset\}\right)$

during this time OPT is moving or idle at origin.

worst case, $t_{move} = t_j - \lambda|\hat{T}|$ $(t_{idle} = 0)$

$$t_{move} \leq \frac{1}{2} Z^{OPT} \text{ (by } \Delta \text{ ineq.)}$$

$$\text{So, } Z^{ALG} = \lambda|\hat{T}| + (t_j - \lambda|\hat{T}|) + |\hat{T}|$$
$$\leq (\lambda + 1.5)|\hat{T}|$$

- It is $(3 + 2/\lambda)$ robust. $(\lambda \in (0,1])$

→ $t_i \leq Z^{OPT}$ $\forall i$ and $|T_{U_t}| \leq Z^{OPT}$ (for any t)

i) if finishes before $\lambda|\hat{T}|$

Then it is following PAH throughout $(Z^{ALG} \leq 2Z^{OPT})$

ii) If can not finish before $\lambda|\hat{T}|$

→ Some unserved request at $\lambda|\hat{T}|$ or something arriving after $\lambda|\hat{T}|$

as it can not finish befor $\lambda|\hat{T}|$, $Z^{OPT} \geq \frac{\lambda|\hat{T}|}{2}$

(as it follows PAH before $\lambda|\hat{T}|$ and PAH is 2 comp)

i) $U_{\lambda|\hat{T}|} \neq \emptyset$, $t_n > (1+\lambda)|\hat{T}|$

follows PAH after $(1+\lambda)|\hat{T}| \Rightarrow Z^{ALG} \leq 2Z^{OPT}$

ii) $U_{\lambda|\hat{T}|} \neq \emptyset$, $t_n < (1+\lambda)|\hat{T}|$

$$Z^{ALG} \leq \lambda|\hat{T}| + |\hat{T}| + |T_{U_{(1+\lambda)|\hat{T}|}}|$$

$$\leq 2Z^{OPT} + \left(\frac{2}{\lambda}\right) Z^{OPT} + Z^{OPT}$$

$$\leq \left(3 + \frac{2}{\lambda}\right) Z^{OPT}$$

iii) $U_{\lambda|\hat{T}|} = \emptyset$ and $t_n > t_j + |\hat{T}|$ (Last service request comes after waiting and moving $\hat{T}$)

PAH is followed after $t_j + |\hat{T}|$

So, $Z^{ALG} \leq 2Z^{OPT}$

iv) $U_{\lambda|\hat{T}|} = \emptyset$ and $t_n \leq t_j + |\hat{T}|$

So, $\lambda|\hat{T}| < t_n < Z^{OPT}$

Server starts last route at $t_j + |\hat{T}|$

So, $Z^{ALG} \leq t_j + |\hat{T}| + |T_{U_{t_j+|\hat{T}|}}|$

$$\leq Z^{OPT} + (1/\lambda) + Z^{OPT}$$

$$\leq (2 + 1/\lambda) Z^{OPT}$$

LAR-NID : $(1.5+\lambda)$ consistent and $(3+2/\lambda)$-robust

LAR-TRUST :

Input: Current time t, the number of requests n, a sequence prediction $\hat{x}$, the set of current released unserviced requests U_t.

- Comput opt $\hat{T} = (\hat{x}_1, \ldots, \hat{x}_n)$ to serve $\hat{x}$ and return to o, where $\hat{x}_i = (\hat{t}_i, \hat{p}_i)$
- Start following $\hat{T}$.

For any $i = 1, 2, \ldots, n$

$\rightarrow$ if $t = t_i$

update $\hat{T}$ by adding x_i after $\hat{x}_i$

$\rightarrow$ if $p(t) = \hat{p}_i$ and $t < t_i$

wait at $\hat{p}_i$ until time t_i (wait until request arrives)

$\rightarrow$ Comp ratio: $1 + 2\varepsilon_{time} + 4\varepsilon_{pos}$

$\Rightarrow$ 1 consistent

$\Rightarrow (2\varepsilon_{time} + 4\varepsilon_{pos})$ - smooth

$\Rightarrow$ not robust

The server gets $\hat{T}$ to follow at the beginning and follows $\hat{T}$. as requests arrive, $\hat{T}$ is modified in 2 ways

i) Server moves to $\hat{p}_i$ but x_i hasn't arrived, waits at $\hat{p}_i$

ii) " adjusts $\hat{T}$ by inserting x_i after $\hat{x}_i$.

- Algo is comp ratio $1 + 2\varepsilon_{time} + 4\varepsilon_{pos}$

T_x opt route for x and $T_{\hat{x}}$ opt route for $\hat{x}$

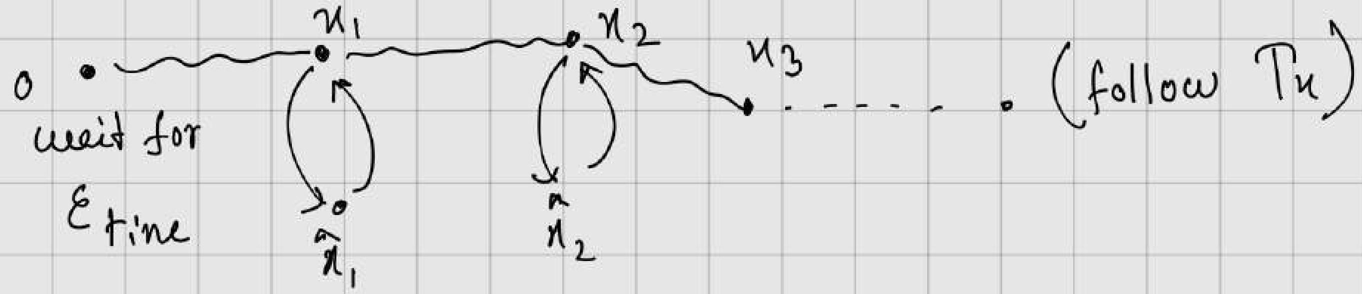
(i) $|T_x| = Z^{OPT}$

(ii) $Z^*_{\hat{x}}$ if requests in $\hat{x}$ are visited following order of T_x

(iii) $|T_{\hat{x}}| = Z'_x$

(iv) Z^{ALG} if " " x " " " " " " $T_{\hat{x}}$

$Z'_{\hat{x}} \leq Z^*_{\hat{x}} \leq Z^{OPT} + \varepsilon_{time} + 2\varepsilon_{pos}$

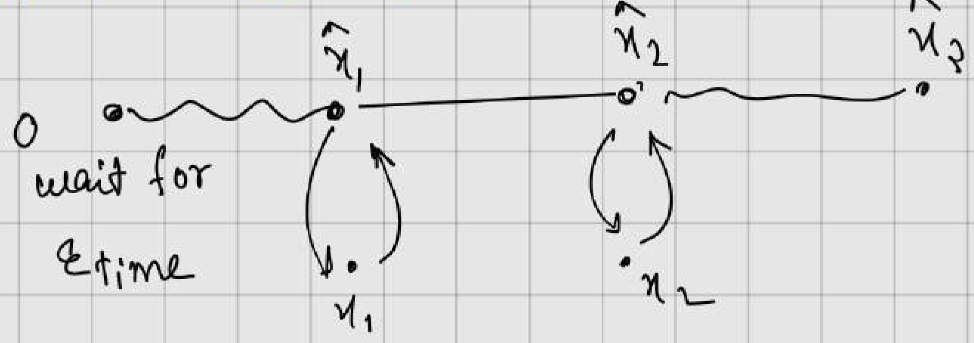


$Z^*_{\hat{x}} \leq Z^{OPT} + \varepsilon_{time} + 2\varepsilon_{pos}$

$Z'_{\hat{x}} \leq Z^*_{\hat{x}}$

optimal

$\# \; Z^{ALG} \leq Z'_{\hat{x}} + \varepsilon_{time} + 2\varepsilon_{pos}$



(LAR-TRUST algo)

$$Z^{ALG} \leq Z'_{\hat{x}} + \varepsilon_{time} + 2\varepsilon_{pos}$$

$$\Rightarrow Z^{ALG} \leq Z^{OPT} + 2\varepsilon_{time} + 4\varepsilon_{pos}$$

LAR-ID :-

Input : Current time t, number of requests n, a seq. of prediction $\hat{x}$, Set

- $F = 0$ (we trust the prediction)
- Compute opt $\hat{T} = (\hat{x}_1, \dots, \hat{x}_n)$ to serve $\hat{x}$ and return to o.
- Start on route $\hat{T}$

1. while $F = 0$ (we trust)
   - if $t = t_i$

   $\rightarrow \hat{T} = (\hat{x}_1, \dots, \hat{x}_i, x_i, \hat{x}_{i+1}, \dots, \hat{x}_n)$

   for $i \in [n]$ (Route update)

   $\rightarrow$ move on route $\hat{T}$ (except for $t = t_n$)

   if $t = t_n$

   $r_1 \longleftarrow$ remaining distance of $\hat{T}$

   $\rightarrow$ Compute $T_{U_{t_n}}$: Start and finish at o and serve U_{t_n}

   $r_2 \longleftarrow d(p(t), o) + |T_{U_{t_n}}|$

   $r_1 > r_2$

   $\rightarrow$ go to o

   $\rightarrow$ $F = 1$ (lost trust)

   $r_2 \geq r_1$

   $\rightarrow$ move on route $\hat{T}$

   - If $p(t) = \hat{p}_i$ and $t < t_i$

   Wait at $\hat{p}_i$ until t_i

2. While $F = 1$

   Follow the route $T_{U_{t_n}}$ to serve U_{t_n}

- Comp. ratio : $\min\left\{3, 1 + (2\varepsilon_{time} + 4\varepsilon_{pos}) / z^{OPT}\right\}$

it is 1-consistent, 3 robust and $(2\varepsilon_{time} + 4\varepsilon_{pos})$- smooth.

- For $r_1 \le r_2$ (Continue on $\hat{T}$ i.e. errors are small)

So it follows LAR- TRUST

$$z^{ALG} \le z^{OPT} + 2\varepsilon_{time} + 4\varepsilon_{pos}$$

- For $r_1 > r_2$

→ better to go back to origin and design a new route

→ returns at $t_n + d(p(t_n), 0)$

Starts to follow TU_{t_n}

$$\begin{aligned} z^{ALG} &= t_n + d(p(t_n), 0) + |TU_{t_n}| \\ &\le t_n + t_n + |TU_{t_n}| \\ &\le Z_{OPT} + Z_{OPT} + Z_{OPT} \\ &= 3 Z_{OPT} \end{aligned}$$

Chooses faster route so $\min\left\{3, 1 + \frac{(2\varepsilon_{time} + 4\varepsilon_{pos})}{Z^{OPT}}\right\}$

T_x is the opt route for requests in x

$T_{\hat{x}}$ " " apprx " " " " " " $\hat{x}$ (using Christofides

- $Z^{OPT} = |T_x|$
- $Z^{*}_{\hat{x}}$ is server visits $\hat{x}$ by following order in T_x
- $Z'_{\hat{x}} = |T_{\hat{x}}|$ " " $T_{\hat{x}}$
- Z^{ALG} " " x " " " " $T_{\hat{x}}$

1. $Z^{*}_{\hat{x}} \leq Z^{OPT} + \varepsilon_{time} + 2\varepsilon_{pos}$
2. $Z'_{\hat{x}} \leq 2.5\, Z^{*}_{\hat{x}}$

$$Z'_{\hat{x}} \leq t_n + |T| \leq 2.5\, Z^{OPT} \leq 2.5\, Z^{*}_{\hat{x}}$$

3. $$\begin{aligned} Z^{ALG} &\leq Z'_{\hat{x}} + \varepsilon_{time} + 2\varepsilon_{pos} \\ &\leq 2.5\, Z^{*}_{\hat{x}} + \varepsilon_{time} + 2\varepsilon_{pos} \\ &\leq 2.5\left(Z^{OPT} + \varepsilon_{time} + 2\varepsilon_{pos}\right) + \varepsilon_{time} + 2\varepsilon_{pos} \\ &= 2.5\, Z^{OPT} + 3.5\, \varepsilon_{time} + 7\varepsilon_{pos} \end{aligned}$$

Case for r_1

For r_2, $$\begin{aligned} Z^{ALG} &= t_n + d(p(t_n), o) + |T_{U_{t_n}}| \\ &\leq Z^{OPT} + Z^{OPT} + 1.5\, Z^{OPT} \\ &= 3.5\, Z^{OPT} \end{aligned}$$

ratio: $\min\left\{3.5,\ 2.5 + \dfrac{3.5\,\varepsilon_{time} + 7\,\varepsilon_{pos}}{Z^{OPT}}\right\}$

Naively,

wait until last request arrives, $Z^{ALG} \leq t_n + |T_x| \leq 2Z^{OPT}$

REDESIGN

→ Input: t, U_t

→ While $U_t \neq 0$

if server is at 0

→ follow route T_{U_t} to serve U_t and return to 0

if server is moving along some $T_{U_{t'}}$ for $t' < t$

→ if a new request arrives go back to 0

$p(t)$ is the pos. of a server by REDESIGN

$d(p(t), 0) \leq \frac{1}{2} Z^{OPT}$ for all t.

3- comp. poly time algo using Christofide's heuristics.

$p(t)$ is the pos. of a server by REDESIGN
$d(p(t), o) \leq \frac{1}{2} z^{OPT}$ for all t.

At time t, dist. b/w server and origin $d(p(t), o)$ is at most distance b/w farthest request and origin and $OPT \geq 2 \times$ farthest request from origin.

Say, $T_{U_{t'}}$ is found by Christofide's $(t' < t)$, it is on the route or on way back. T^* is optimal route to visit set of requests in $T_{U_{t'}}$. (Neither $T_{U_{t'}}$ nor T^* considers release time of requests)

i) On route $T_{U_{t'}}$

a and b is p_i for some request or origin o

$$|T^*| \geq d(o, a) + d(a, b) + d(b, o)$$

$$d(p(t), o) \leq \min \{ d(p(t), a) + d(a, o),\ d(p(t), b) + d(b, o) \}$$
$$\leq \frac{1}{2} |T^*| \leq \frac{1}{2} z^{OPT}$$

ii) On route back home after terminating a route $T_{U_{t'}}$ then a request $x_i = (t_i, p_i)$ came b/w t' and t

$d(p(t_i), o) \leq \frac{1}{2} z^{OPT}$ (by above)

$$\Rightarrow d(p(t), o) \leq d(p(t_i), o) \leq \frac{1}{2} z^{OPT}$$

REDESIGN is a 3-comp. poly-time algo for OLTSP in a metric space using Christofides.

The server recieves last request x_n, goes to 0 and finds route $T_{U_{t_n}}$ by Christofides.

$$Z^{ALG} = t_n + d(p(t_n), 0) + |T_{U_{t_n}}|$$

$t_n \leq Z^{OPT}$

$|T_{U_{t_n}}| \leq 1.5\, T^* \leq 1.5\, Z^{OPT}$

$d(p(t_n), 0) \leq 0.5\, Z^{OPT}$

$\Rightarrow Z^{ALG} \leq 3 Z^{OPT}$

LAR-LAST

Input: $t, U_t, \hat{t}_n$

while $U_t \neq \emptyset$

- if Server is at o
  i.e. $p(t) = 0$
  → Compute T_{U_t}
  → If $t < t_n$ and $t + |T_{U_t}| > \hat{t}_n$
    → find t_{back} s.t. $t_{back} + d\left(p(t_{back}), 0\right) = \hat{t}_n$
    → Compute T'_{U_t} by asking the server to go to origin o at t_{back} along shortest path.
    → Follow T'_{U_t}
  → o/w follow T_{U_t}

- If Server is on $T_{U_{t'}}$ for some $t' < t$ then
  if $x_i = (t_i, p_i)$ comes (Redesign)
  go to origin o

LAR-LASP is a $\min\{4, 2.5 + |\varepsilon_{last}|/z^{OPT}\}$ - comp poly-time algo where $\varepsilon_{last} = \hat{t}_n - t_n$

(2.5 consistent, 4 robust and $|\varepsilon_{last}|$ -smooth)

ⓐ $\hat{t}_n \leq t_n$

$$z^{ALG} = t_n + d(p(t_n), 0) + |T_{U_{t_n}}|$$
$$\leq z^{OPT} + 0.5\, z^{OPT} + 1.5\, z^{OPT}$$
$$= 3\, z^{OPT}$$

also, $d(p(t_n), 0) \leq t_n - \hat{t}_n = -\varepsilon_{last} = |\varepsilon_{last}|$ (as $\varepsilon_{last} \leq 0$)

$$z^{ALG} \leq 2.5\, z^{OPT} + |\varepsilon_{last}|$$

ⓑ $\hat{t}_n > t_n$

$$z^{ALG} \leq \hat{t}_n + |T_{U_{\hat{t}_n}}|$$

t_L is last time before $\hat{t}_n$ a route T^L is planned

T'^L is the new route if T^L is adjusted ($T^L = T'^L$ if T^L can return before $\hat{t}_n$)

So, $t_L + |T'^L| \leq \hat{t}_n$

i) If $t_L + |T^L| \leq \hat{t}_n$

$z^{ALG} \leq \hat{t}_n$ (No readjustment)

So, we're done by $\hat{t}_n$ and it is same as REDESIGN

So, $z^{ALG} \leq 3 z^{OPT}$

ii) If $t_L + |T^L| > \hat{t}_n$ and $t_n \leq t_L$

↳ Too long route

↳ Algo finds t_{back} and alternative route T'^L

↳ Server is at o at $\hat{t}_n$

↳ We could have continued with T^L but going back at t_{back} makes us incur a cost of at most $2d(p(t_{back}), o)$

$$\text{So, } Z^{ALG} \leq 3Z^{OPT} + 2d(p(t_{back}), o) \qquad \left(d(p(t), o) \leq \tfrac{1}{2} Z^{OPT}\right)$$
$$\leq 4Z^{OPT}$$

iii) If $t_L + |T^L| > \hat{t}_n$ and $t_n > t_L$

- too long route
- at least one request is unknown at t_L
- Server returns at $\hat{t}_n$ and starts the last route $|T_{U_{\hat{t}_n}}|$

$$Z^{ALG} \leq \hat{t}_n + |T_{U_{\hat{t}_n}}|$$
$$= t_L + |T'^L| + |T_{U_{\hat{t}_n}}|$$
$$\leq t_n + |T'^L| + |T_{U_{\hat{t}_n}}|$$
$$\leq Z^{OPT} + 1.5\,Z^{OPT} + 1.5\,Z^{OPT}$$
$$= 4Z^{OPT}$$

Another analysis

$$z^{ALG} \le \hat{t}_n + |T_{U_{\hat{t}_n}}|$$

$$= t_n + \varepsilon_{last} + |T_{U_{\hat{t}_n}}|$$

$$\le Z_{OPT} + \varepsilon_{last} + 1.5\, Z_{OPT}$$

$$= 2.5\, Z_{OPT} + \varepsilon_{last}$$

So, $z^{ALG} \le \min\{4 Z^{OPT}, 2.5\, Z^{OPT} + |\varepsilon_{last}|\}$