E0 206 : Homework 3

Due date : 19/11/20

Instructions

- All problems carry equal weight.
- You are forbidden from consulting the internet. You are strongly encouraged to work on the problems on your own.
- You may discuss these problems with your group (at most 3 people including you). However, you must write your own solutions and list your collaborators for each problem.

1. Consider $G := (V, E)$ be a simple graph, where each vertex $v \in V$ is associated with a list $S(v)$ of colors. Here, $|S(v)| \geq 10d$, where $d \geq 1$. Also for each $v \in V$, and $c \in S(v)$ there are at most d neighbors u of v such that $c \in S(u)$. Prove that there is a proper coloring of G where each vertex v is assigned a color from its list $S(v)$.

2. The diameter of a graph is the maximum length of the shortest path between a pair of nodes. Let $G \in \mathcal{G}(n, p)$ be a random graph, where $p = c\sqrt{(\ln n/n)}$. Show that the graph almost surely has diameter > 2 for $c < \sqrt{2}$, and the graph almost surely has diameter ≤ 2 for $c > \sqrt{2}$.

3. A fair coin is being tossed at the same time when a fair die (six-faced) is being rolled. Let C be the the outcome of the coin toss, i.e., $C \in \{0, 1\}$ with equal probability. Let $D \in [6]$ be the number on the upper face of the die. Let us define random variable $X = C + D$ and $Y = D - C$, respectively.

   Calculate the entropies $H(X), H(Y)$, the conditional entropies $H(X|Y), H(Y|X)$, the joint entropy $H(X, Y)$, and the mutual information $I(X; Y)$.

4. Assume you are given a deck of n cards, sorted in increasing order $1, 2, \ldots, n$. A card is selected uniformly at random. Then it is removed and inserted again at one of the n available positions uniformly at random. What is the entropy of the resulting deck?