E0 206 : Homework 6

Due date : 02/01/21

Instructions

- All problems carry equal weight.
- You are forbidden from consulting the internet. You are strongly encouraged to work on the problems on your own.
- You may discuss these problems with your group (at most 3 people including you). However, you must write your own solutions and list your collaborators for each problem.

1. For each of the following, either prove that the problem is in P or that it is NP-hard.
   (a) Given an undirected graph $G = (V, E)$ and vertices $a, b \in V$, compute the following.
   $$\min_{x_1,\ldots,x_{|V|}\in\mathbb{R}} \sum_{\{i,j\}\in E} |x_i - x_j| \qquad \text{subject to} \qquad x_i \in [0,1]\ \forall i \in V,\ x_a = 0, x_b = 1.$$
   (b) Given an undirected graph $G = (V, E)$, compute the following.
   $$\max_{x_1,\ldots,x_{|V|}\in\mathbb{R}} \sum_{\{i,j\}\in E} |x_i - x_j| \qquad \text{subject to} \qquad x_i \in [0,1]\ \forall i \in V.$$
2. Let $G \sim \mathcal{G}(n, 1/2)$ be a random graph constructed as follows. Starting with a set of n vertices, an edge is added between each pair of vertices independently with probability $1/2$. Prove that $\phi_G = \Omega(1)$ with high probability.
3. Let $G = (V, E)$ be a d-regular β spectral expander, and let A be its adjacency matrix.
   (a) Let $\{e_i\}_{i\in[n]}$ be the standard basis vectors. Prove that if $e_a^T A^l e_b > 0\ \forall a, b \in V$, then the diameter of G is at most l.
   (b) Give an upper bound on the diameter of G in terms of n, d and β.
4. (O'Donnell) problem 1.8

Optional practice problems: (will not be graded or provided with solutions)

1. (O'Donnell) all relevant problems from chapter 1.