E0 206 : Homework 7

Due date : 15/01/21

Instructions

- All problems carry equal weight.
- You are forbidden from consulting the internet. You are strongly encouraged to work on the problems on your own.
- You may discuss these problems with your group (at most 3 people including you). However, you must write your own solutions and list your collaborators for each problem.

1. (Vishnoi) exercise 3.8
2. (Vishnoi) exercise 5.4
3. Let $C_n = \{A \in \mathbb{R}^{n \times n}$ such that $A \succeq 0\}$. Prove that C_n is a convex set.
4. Recall the zero-sum game played between players R and C where $A \in \mathbb{R}^{m \times n}$. The von Neumann's minimax theorem says that $\max_x \min_y x^T Ay = \min_y \max_x x^T Ay$, where the max and min are taken over mixed strategies for the players. We will now prove this.

   Let $A_1, \ldots, A_n$ be the columns of A.

   (a) Prove that the optimal value of the following LP is equal to $\max_x \min_y x^T Ay$.

   $$\begin{aligned} & \max t \\ \text{s. t.} \quad & \langle u, A_i \rangle \geq t \qquad \forall i \in [n] \\ & \sum_{j \in [m]} u_j = 1 \\ & u_j \geq 0 \qquad \forall j \in [m] \end{aligned}$$

   (b) Write the dual of the above LP.

   (c) Prove that the optimal value of the dual LP is equal to $\min_y \max_x x^T Ay$.

   (d) Prove von Neumann's minimax theorem.

Optional practice problems: (will not be graded or provided with solutions)

Relevant problems from (Vishnoi).