CSA E0 221: Discrete Structures	November 19, 2014
Chalk & Talk Session	
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1 Chordal Graph :

Definition 1 *A graph is called chordal if every cycle with more than three vertices has an edge connecting two non-consecutive vertices or has an edge that is not part of the cycle but connects two vertices of the cycle.*

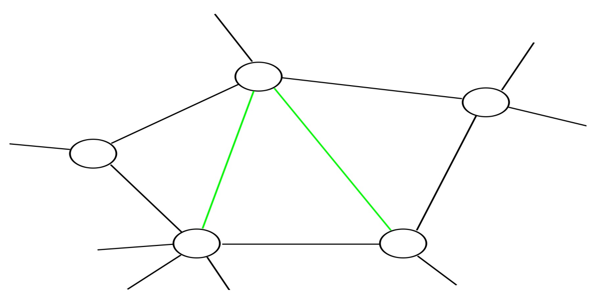
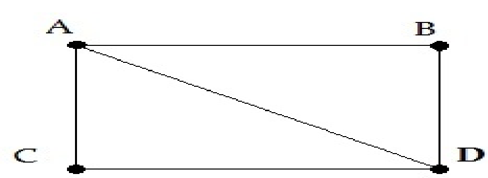


Figure 1: Chordal graph

Why Chordal Graphs are important ?

- Chordal graphs are a subset of the perfect graphs.
- They may be recognized in polynomial time, and several problems that are hard on other classes of graphs such as graph coloring may be solved in polynomial time when the input is chordal graph.
- All induced subgraphs of a chordal graph are chordal.
- Chordal graphs are also called triangulated and perfect elimination graphs.

Definition 2 *In a graph G, a vertex v is called simplicial if and only if the subgraph of G induced by the vertex set* $\{v\} \cup N(v)$ *is a complete graph.*



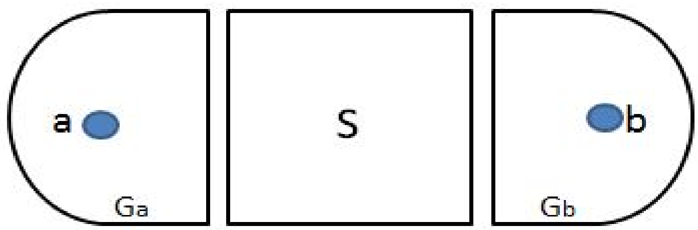
in the graph above, vertex A is simplicial, while vertex D is not.

Definition 3 *A graph G on n vertices is said to have a perfect elimination ordering if and only if there is an ordering $\{v_1, ...v_n\}$ of Gs vertices, such that each vi is simplicial in the subgraph induced by the vertices $\{v_1, ...v_i\}$. As an example, the graph above has a perfect elimination ordering, witnessed by the ordering (2, 1, 3, 4) of its vertices.*

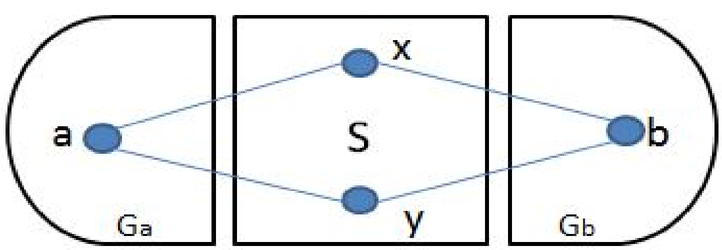
Theorem 1 *(Dirac, 1961) Each chordal graph has a simplicial vertex and if G is not a clique it has two non adjacent simplicial vertices.*

Proof

- If G is a clique, done.
- If not, assume that it has two non-adjacent nodes a and b and the lemma is true to all graphs with fewer vertices than G. Let S be a minimal vertex separator for a and b. Let G_a and G_b be the connected components of a and b respectively.



- If S is not a clique, then there exists x and y such that there is no edge between them, and because S is minimal vertex separator there is a cycle $\{a, \dots, x, \dots, b, \dots, y, \dots, a\}$. It is easy to show that there exist a minimal cycle with length no less than 4 with no chords in it, contradiction.



- S is a clique. $G_a + S$ is smaller than G therefore by induction the lemma holds, i.e. $G_a + S$ is a clique or has two non adjacent simplicial vertices, one of each must be in G_a. Any simplicial vertex in G_a is a simplicial vertex in G because all elements of Adj(a) are inside $G_a + S$. Thus from G_a and G_b we get two simplicial vertices in G.

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