

## Discrete Structures

### Problem Set 2

#### 2.1 Havel-Hakimi Criteria (\*)

Give a realization for the degree sequence  $(3, 3, 2, 2, 2, 2, 2, 2)$  that cannot be obtained using Havel-Hakimi Criterion.

#### 2.2 Non-isomorphic Realizations of Degree Sequence (\*)

Construct two *non-isomorphic* realizations of the degree sequence  $(2, 2, 2, 1, 1)$  using Havel-Hakimi Criterion.

#### 2.3 Graph Realization

Let  $n \geq 1$  be an integer.

- a) (\*) Does there exist a realization for the sequence  $(n-1, n-1, n-2, \dots, 4, 3, 2, 1)$ ?
- b) (\*\*) Does there exist a realization for the sequence  $(n, n, n-1, n-1, \dots, 3, 3, 2, 2, 1, 1)$ ?

#### 2.4 Graphic Sequence for Regular Graph (\*)

Show that a regular sequence  $(d, d, \dots, d)$  of length  $n$  is graphic if and only if  $d \leq n-1$ , and  $d \cdot n$  is even.

#### 2.5 Graphic Sequence for graph (\*\*)

Show that any sequence  $S = (d_1 \geq d_2 \geq \dots, \geq d_n)$  of non-negative integers is a degree sequence for some graph (not necessarily simple) if and only if  $\sum_{i=1}^n d_i$  is even.

#### 2.6 Graphic Sequence for loopless Graph (\*\*)

Show that any sequence  $S = (d_1 \geq d_2 \geq \dots, \geq d_n)$  of non-negative integers is a degree sequence for some loopless graph (it can have multiple edges) if and only if  $\sum_{i=1}^n d_i$  is even and  $d_1 \leq d_2 + d_3 + \dots + d_n$ . (Hint. Use induction for the sufficiency)

Good Luck!