IISc, CSA   Arpita Patra
28 October'2014

Discrete Structures
Problem Set 7

7.1 Pr*ü*fer Code

a) (∗) Find the tree whose Prufer code is $(k, k, \ldots, k)$ (k is repeated $n - 1$ times) for some k.

b) (∗) Find the tree whose Prufer code is $(a, b, a, b \ldots, a, b)$ (a and b is repeated $n - 1$ times) for some a and b.

7.2 Eulerian Graphs

a) (∗) Does there exists a simple Eulerian graph on even number of vertices and odd number of edges?

b) (∗) If G is Eulerian, then show that $k_1(G)$ is even.

c) (∗∗) A connected graph G is Eulerian if and only if its edge set can be partitioned into cycles.

d) (∗∗) A connected graph is Eulerian if and only if each of its edges lies on an odd number of cycles.

e) (∗∗) The number of edge-disjoint paths between any two vertices of an Euler graph is even.

7.3 Super-Eulerian Graphs (∗∗)

A non-Eulerian graph G is said to be super-Eulerian if it has a spanning Eulerian subgraph. Show that: If G is any connected graph and if each edge of G belongs to a triangle in G, then G has a spanning Eulerian subgraph.

7.4 Line Graph of an Eulerian Graph (∗∗)

Let H be an simple graph with n vertices $v_1, v_2, \ldots, v_n$ and m edges $e_1, \ldots, e_m$. The line graph $L(H)$ of H is a simple graph with the verices $e_1, \ldots, e_m$ in which e_i and e_j are adjacent iff they are adjacent in H.

If G is a connected simple graph, then show that $L(G)$ is Eulerian iff either every vertex in G has even degree or every vertex has odd degree.

Good Luck!