

Discrete Structures

Problem Set 9

9.1 Vertex Chromatic No. and Independence No. (**)

For every graph show that $\frac{n(G)}{\chi(G)} \leq \frac{n(G)}{\alpha_0(G)} \leq \chi_0(G) \leq n + 1 - \alpha_0(G)$, where \overline{G} denotes the complement graph of G .

9.2 Vertex Chromatic No. of Sum graphs (**)

Prove that $\chi_0(G_1 + G_2) = \chi_0(G_1) + \chi_0(G_2)$.

9.3 k -critical Graphs (**)

A graph G is called k -critical if (i) $\chi_0(G) = k$ and (ii) $\chi_0(H) < k$ for every proper sub-graph of G . If G is k -critical then show that:

- a) G is connected.
- b) $\delta(G) \geq k - 1$.
- c) G has no pair of sub-graphs G_1 and G_2 such that $G = G_1 \cup G_2$ and $G_1 \cap G_2$ is a complete graph.
- d) $G - v$ is connected for every $v \in V(G)$ when $k > 1$.

9.4 Graph with $\chi_0(G) = k$ (**)

Show that every graph with $\chi_0(G) = k$ contains at least $k(k - 1)/2$ edges.

Good Luck!