Computational complexity: Assignment 2

Due date: September 30, 2013

General instructions:

- Write your solutions by furnishing all relevant details (you may assume the results already covered in the class).
- You are strongly urged to solve the problems by yourself.
- If you discuss with someone else or refer to any material (other than the course notes) then please put a reference in your answer script stating clearly whom or what you have consulted with and how it has benifited you. We would appreciate your honesty.
- If you need any clarification, please ask the instructor.

Total: 50 points

In the following problems, n stands for length of input string (unless mentioned otherwise in the problem).

1. (**5 points**) [Excercise-2.18 from Arora-Barak's book] Prove that the language HAMPATH of undirected graphs with Hamiltonian paths is NP-complete. Prove that the Travelling Salesman Problem (TSP) is NP-complete. Prove that the language HAMCYCLE of undirected graphs that contain Hamiltonian cycle (a simple cycle involving all vertices) is NP-complete.

2. (**5 points**) [Excercise-2.34 from Arora-Barak's book] Suppose that you are given a graph G and a number k and are told that either (i) the smallest vertex cover of G is of size k or (ii) it is of size at least $3k$. Show a polynomial-time algorithm that can distinguish between these two cases. Can you do it with a smaller constant than 3? Since VERTEX COVER problem is NP-hard, why does this algorithm not show that $\mathsf{P} = \mathsf{NP}$?

3. (**6 points**) [Excercise-3.1 from Arora-Barak's book] Show that the following language is undecidable:

   $$\{\underline{M} : M \text{is a machine that runs in } O(n^2) \text{ time}\}.$$

   Here $\underline{M}$ is the Turing machine M's representation as a binary string.

4. (**5 points**) [Excercise-3.2 from Arora-Barak's book] Show that $\mathsf{SPACE}(n) \neq \mathsf{NP}$.

5. (**5 points**) [Excercise-3.3 from Arora-Barak's book] Show that there is a language $B \in \mathsf{EXP}$ such that $\mathsf{P}^B \neq \mathsf{NP}^B$.

6. (**6 points**) [Excercise-4.4 & 4.5 from Arora-Barak's book] Show that the following languages are in NL:

   (a) (**3 points**) $\{\underline{G} : G \text{ is a strongly connected digraph}\}$.

   (b) (**3 points**) 2SAT.

7. (**5 points**) [Excercise-4.7 from Arora-Barak's book] Prove that in the certificate definition of NL, if we allow the verifier machine to move its head back and forth on the certificate then the class being defined changes to NP.

8. (**4 points**) [Excercise-4.12 from Arora-Barak's book] Define polyL to be $\cup_{c>0}\mathsf{SPACE}(\log^c n)$. Steve's Class SC (named in honor of Steve Cook) is defined to be the set of languages that can be decided by deterministic machines that run in polynomial time and $\log^c n$ space for some $c > 0$. It is an open problem whether PATH $\in \mathsf{SC}$. Why does Savitch's theorem not resolve this question? Is SC the same as $\mathsf{polyL} \cap \mathsf{P}$.

9. (**Bonus problem**) [Excercise-3.9 from Arora-Barak's book] Suppose we pick a random language C by choosing every string to be in C independently with probability $1/2$. Prove that with probability one $\mathsf{P}^C \neq \mathsf{NP}^C$.