**E0 224: Computational Complexity Theory**
**Indian Institute of Science**

**Due date: September 21, 2017** **Total points: 50**

1. (a) (**2 points**) We define the class Quasi-P as follows:

$$\text{Quasi-P} = \bigcup_{i \geq 1} \text{DTIME}(n^{(\log n)^i}).$$

   Show that if $L_1 \leq_p L_2$ and $L_2 \in$ Quasi-P then $L_1 \in$ Quasi-P.

   (b) (**4 points**) Let $G = (V, E)$ be a graph, and suppose there is an algorithm that tests in polynomial time if $G$ has a Hamiltonian cycle or not. Show that there exists a polynomial time algorithm that outputs a Hamiltonian cycle in $G$, if one exists.

   (c) (**2 points**) Let $L_1, L_2 \in$ NP. Are $L_1 \cup L_2$ and $L_1 \cap L_2$ also in NP?

   (d) (**3 points**) Let $L_1, L_2 \in \text{NP} \cap \text{co-NP}$. Show that $L_1 \oplus L_2 \in \text{NP} \cap \text{co-NP}$, where $L_1 \oplus L_2 := \{x : x \text{ is in exactly one of } L_1, L_2\}$.

2. (**5 points**) Consider the following problem.

$$k\text{-Col} := \{G = (V, E) : G \text{ has a coloring with } k \text{ colors}\},$$

   where a coloring of $G$ with $k$ colors is an assignment of a number in $\{1, \ldots, k\}$ to each vertex such that no two adjacent vertices get the same color. For which value(s) of $k \in \mathbb{N}$ is $k$-Col in P? Justify your answer.

3. (**9 points**) Suppose that there are $n$ boolean variables $y_1, \ldots, y_n$ and $r$ clauses such that each clause contains at most 2 literals and $m \in \mathbb{N}$, $m \leq r$. Show that deciding if there exists an assignment that satisfy $m$ out of these $r$ clauses is NP-complete.

4. (**Bonus question: 6 points**) Prove that NP $\neq$ DSPACE($n$).

5. (**4 points**) Give an example of a function which is not time constructible.

6. (a) (**2 points**) Show that coPSPACE = PSPACE.

   (b) (**3 points**) Show that coNP $\subseteq$ PSPACE.

7. (**8 points**) Show that TQBF is PSPACE-complete even under log-space reductions.

8. (**8 points**) A directed graph $G = (V, E)$ is strongly connected if for every two nodes $u, v \in V$ there is path from $u$ to $v$ and from $v$ to $u$ in $G$. Show that the following language is NL-complete,

$$\{G \mid G \text{ is a strongly connected directed graph}\}.$$