E0 224: Computational Complexity Theory
Indian Institute of Science
Assignment 3

Due date: Dec 5, 2017 **Total points: 50**

1. (**8 marks**)

   (a) A boolean formula ϕ is a 3 DNF if ϕ can be written as $\phi = m_1 \vee m_2 \vee \cdots \vee m_r$, $r \in \mathbb{N}$, where the i-th term $m_i = (x_{i1} \wedge x_{i2} \wedge x_{13})$ has at most 3 literals for $i \in \{1, \ldots, r\}$. Let

   $$\#3DNF : \{\phi \mid \phi \text{ is a 3 DNF}\} \to \mathbb{N},$$

   where $\#3DNF(\phi)$ is the number of satisfying assignments of ϕ. Show that if $\#3DNF \in FP$ then $P = NP$.

   (b) Give a polynomial time algorithm that checks whether a given bipartite graph $G = (V, E)$ is contained in $\oplus$Perfect Matchings, where $\oplus$Perfect Matchings is the set of all bipartite graphs having odd number of perfect matchings.

2. (**9 marks**)
   Show that the following problem is #P complete.
   Imperfect Matching:
   Input: Bipartite graph $G = (V, E)$, where $|V| = 2m$ and each partition has m vertices.
   Output: Number of matchings (not necessarily perfect) in G.

3. (**7 marks**)
   Show that if there is a polynomial time algorithm that approximates #CYCLE within a factor $\frac{1}{2}$ then P=NP.

4. Let $r(n) = o(\log n)$. Show the following

   (a) (**5 marks**) If SAT $\in$ PCP$(r(n), 1)$ then P = NP.

   (b) (**7 marks**) If NP $\subseteq$ PCP$(r(n), r(n))$ then P=NP.

5. (**6 marks**) If P = NP then show for $f \in \#P$ there exists a deterministic polynomial time algorithm that approximates f to within a factor of $1 \pm \epsilon$ for arbitrarily small constant $\epsilon > 0$.

6. (**8 marks**) Consider the following problem: Given a system of linear equations in n with coefficients that are rational numbers, determine the largest subset of equations that are simultaneously satisfiable. Show that there is a constant $\rho < 1$ such that approximating the size of this subset is NP-hard.