EO 224: Computational complexity theory - Assignment 1

Due date: September 28, 2018

General instructions:

- Write your solutions by furnishing all relevant details (you may assume the results already covered in the class).
- You are strongly urged to solve the problems by yourself.
- If you discuss with someone else or refer to any material (other than the class notes) then please put a reference in your answer script stating clearly whom or what you have consulted with and how it has benifited you. We would appreciate your honesty.
- If you need any clarification, please contact the instructor.

Total: 50 points

1. (**3 points**) Let $L_1, L_2 \in \mathsf{NP}$. Are $L_1 \cup L_2$ and $L_1 \cap L_2$ also in NP?
2. (**4 points**) Let $L_1, L_2 \in \mathsf{NP} \cap \mathsf{co\text{-}NP}$. Show that $L_1 \oplus L_2 \in \mathsf{NP} \cap \mathsf{co\text{-}NP}$, where $L_1 \oplus L_2 := \{x : x \text{ is in exactly one of } L_1,\, L_2\}$.
3. (**5 points**) Let QUADEQ be the language of all satisfiable sets of quadratic equations over 0/1 variables (a quadratic equation over $u_1, \ldots, u_n$ has the form $\sum_{i,j \in [n]} a_{i,j} u_i u_j + \sum_{i \in [n]} a_i u_i = b$) where addition is modulo 2. Show that QUADEQ is NP-complete.
4. (**5 points**) Prove that the function $H(n)$ defined in the proof of Ladner's theorem is computable in time polynomial in n.
5. (**7 points**) Prove that in the certificate definition of NL, if we allow the verifier machine to move its head back and forth on the certificate then the class being defined changes to NP.
6. (**8 points**) A directed graph $G = (V, E)$ is strongly connected if for every two nodes $u, v \in V$ there is path from u to v and from v to u in G. Show that the following language is NL-complete,
$$\{G \mid G \text{ is a strongly connected directed graph}\}.$$
7. (**8 points**) Prove that 2-SAT is NL-complete.
8. (**10 points**) A language L is *sparse* if there exists a constant c such that for every integer $n \geq 0$, the number of strings of length n belonging to L is bounded by n^c. Show that if a sparse language is NP-complete then $\mathsf{P} = \mathsf{NP}$.