EO 224: Computational complexity theory - Assignment 2

Due date: November 22, 2018

General instructions:

- Write your solutions by furnishing all relevant details (you may assume the results already covered in the class).
- You are strongly urged to solve the problems by yourself.
- If you discuss with someone else or refer to any material (other than the class notes) then please put a reference in your answer script stating clearly whom or what you have consulted with and how it has benifited you. We would appreciate your honesty.
- If you need any clarification, please contact the instructor.

Total: 50 points

1. (**18 points**) Prove the following:
   (a) (**5 points**) If $\mathsf{BPP} = \mathsf{PP}$ then PH collapses.
   (b) (**5 points**) PP has a complete problem under polynomial time Karp reduction.
   (c) (**5 points**) PP is closed under complementation and unions.
   (d) (**3 points**) $\mathsf{PP} \subseteq \mathsf{PSPACE}$.
2. (**6 points**) Let BPL be the logspace variant of BPP i.e. a language L is in BPL if there is an $O(\log(n))$ space probabilistic Turing machine M such that $\Pr[M(x) = L(x)] \geq 2/3$. Prove that $\mathsf{BPL} \subseteq \mathsf{P}$.
3. (**6 points**) Prove that computing a n-th order partial derivative of a polynomial is $\#\mathsf{P}$-hard if the input polynomial is given in the form $\prod_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_j$, where a_{ij} are integers.
4. (**8 points**) Prove that #2-SAT is $\#P$-complete.
5. (**4 points**) Give a polynomial time algorithm that checks whether a given bipartite graph $G = (V, E)$ is contained in $\oplus$Perfect Matchings, where $\oplus$Perfect Matchings is the set of all bipartite graphs having odd number of perfect matchings.
6. (**8 points**) Consider the following problem: Given a system of linear equations in n variables with coefficients that are rational numbers, determine the largest subset of equations that are simultaneously satisfiable. Show that there is a constant $\rho < 1$ such that approximating the size of this subset is NP-hard.