

E0 224: Computational Complexity Theory
Indian Institute of Science
Assignment 3

Due date: Nov 20, 2022

Total marks: 50

1. **(4 marks)** Give a polynomial time algorithm that checks whether a given bipartite graph $G = (V, E)$ is contained in $\oplus\text{Perfect Matchings}$, where $\oplus\text{Perfect Matchings}$ is the set of all bipartite graphs having odd number of perfect matchings.
2. **(5 marks)** Prove that for any $n \times n$ matrix $A = (a_{i,j})_{i,j \in [n]}$,

$$\text{perm}(A) = \sum_{S \subseteq [n]} (-1)^{n-|S|} \prod_{i \in [n]} \left(\sum_{j \in S} a_{i,j} \right).$$

Use this to design an algorithm to compute the permanent in time $2^n \cdot \text{poly}(n)$.

3. **(4 marks)** Consider the following problem: Given an n -variate polynomial f in the form $\prod_{i \in [n]} \sum_{j \in [n]} a_{i,j} x_j$, where $a_{i,j}$ are integers, and $e_1, \dots, e_n \in \mathbb{Z}_{\geq 0}$ s.t. $e_1 + \dots + e_n = n$, compute

$$\frac{\partial^n f}{\partial x_1^{e_1} \partial x_2^{e_2} \dots \partial x_n^{e_n}}.$$

Prove that the problem is $\#P$ -hard.

4. **(6 marks)** Prove that $\text{ZPP} = \text{RP} \cap \text{co-RP}$.
5. **(6 marks)** Let BPL be the logspace variant of BPP, i.e., a language L is in BPL if there is an $O(\log(n))$ space probabilistic Turing machine M such that $\Pr[M(x) = L(x)] \geq 2/3$. Prove that $\text{BPL} \subseteq \text{P}$.
6. **(7 marks)** Prove that BP.NP is in Σ_3 .
7. **(9 marks)** Prove that $\overline{\text{SAT}} \in \text{BP.NP}$ implies $\text{PH} = \Sigma_3$.
8. **(9 marks)** Give a randomized algorithm that takes input two $n \times n$ matrices A and B with integer entries and does the following: If A and B are similar, then with high probability the algorithm outputs an $n \times n$ invertible matrix C with rational entries such that $CAC^{-1} = B$; otherwise it outputs ‘ A not similar to B ’. Ensure that your algorithm runs in polynomial time.