E0 224: Computational Complexity Theory
Indian Institute of Science
Assignment 3

Due date: Nov 17, 2025 **Total marks: 50**

General instructions:

- Write your solutions furnishing all relevant details. You may assume the results proved in the class.
- You are encouraged to solve the problems yourself.
- If you discuss with someone or consult material (other than course lecture notes), then please ensure that you understand the solutions completely before writing them in our own language. Please put appropriate citations stating clearly whom or what you have consulted and how it has benefited you.
- If you need any other clarification, please contact the instructor.

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1. (**4 marks**) Give a polynomial time algorithm that checks whether a given bipartite graph $G = (V, E)$ is contained in $\oplus$Perfect Matchings, where $\oplus$Perfect Matchings is the set of all bipartite graphs having odd number of perfect matchings.

2. (**5 marks**) Prove that for any $n \times n$ matrix $A = (a_{i,j})_{i,j \in [n]}$,
$$\mathrm{perm}(A) = \sum_{S \subseteq [n]} (-1)^{n-|S|} \prod_{i \in [n]} \left(\sum_{j \in S} a_{i,j} \right).$$
Use this to design an algorithm to compute the permanent in time $2^n \cdot \mathrm{poly}(n)$.

3. (**4 marks**) Consider the following problem: Given an n-variate polynomial f in the form $\prod_{i\in[n]} \sum_{j\in[n]} a_{i,j} x_j$, where $a_{i,j}$ are integers, and $e_1, \ldots, e_n \in \mathbb{Z}_{\geq 0}$ s.t. $e_1 + \ldots + e_n = n$, compute
$$\frac{\partial^n f}{\partial x_1^{e_1} \partial x_2^{e_2} \cdots \partial x_n^{e_n}}.$$
Prove that the problem is $\#\mathsf{P}$-hard.

4. (**6 marks**) Prove that $\mathsf{ZPP} = \mathsf{RP} \cap \mathsf{co} - \mathsf{RP}$.

5. (**6 marks**) Let BPL be the logspace variant of BPP, i.e., a language L is in BPL if there is an $O(\log(n))$ space probabilistic Turing machine M such that $\Pr[M(x) = L(x)] \geq 2/3$. Prove that $\mathsf{BPL} \subseteq \mathsf{P}$.

6. (**7 marks**) Prove that $\mathsf{BP.NP}$ is in Σ_3.

7. (**9 marks**) Prove that $\overline{SAT} \in \mathsf{BP.NP}$ implies $\mathsf{PH} = \Sigma_3$.

8. (**9 marks**) Give a randomized algorithm that takes input two $n \times n$ matrices A and B with integer entries and does the following: If A and B are similar, then with high probability the algorithm outputs an $n \times n$ invertible matrix C with rational entries such that $CAC^{-1} = B$; otherwise it outputs 'A not similar to B'. Ensure that your algorithm runs in polynomial time.