UMC 201: Data Structures & Algorithms

Lecture 1: Course info; Introduction; Insertion sort; Order notations

Indian Institute of Science

Course Info

- **Course no.:** UMC 201 **Credits:** 3:1
- **Instructors:** Chandan Saha (chandan@iisc)
  Sathish Govindarajan (gsat@iisc)
- **Lecture time:** T,Th, 10-11:20 am **Venue:** G-21
- **Teams group:** Will be set up soon.
- **TAs:** Agrim Dewan (agrimdewan@iisc); Sreesankar T M (sreesankar1@iisc), and 4 others.
- **Course homepage:**
  https://www.csa.iisc.ac.in/~chandan/courses/dsa26/home.html

Course Info

- **Prerequisites**: UENG 101; Mathematical maturity.

- **Primary reference:** Introduction to Algorithms by *Cormen, Leiserson, Rivest and Stein*. (We'll follow this textbook ***closely***.)

- **Other references:** Data Structures and Algorithm Analysis in C by *Weiss*; Algorithm Design by *Kleinberg & Tardos*.

Course Info

- **Number of lectures:** ~25.

- Chandan Saha will teach the *first half* of the course (until the mid-term exam). Sathish Govindarajan will teach the *second half*.
- **Lectures:** Slides of the *first half* of the course will be posted on the course homepage. The *second half* will have board lectures; materials will be posted on Teams as and when required.

Course Info

- **Grading policy**: Two *class tests* - 30% (15% each)
  Mid-term exam - 35%
  End-term exam - 35%

- **Mid-term exam:** Between Sep 19 – Sep 26
- **Final-term exam:** Between Nov 20 – Dec 2

Class Tests

- *Practice problems* will be provided in the slides (in the first half) and over Teams (in the second half).
- In a class test, a student will be asked to solve problems (which will be chosen from among the practice problems or slight variants of them).
- There will be multiple sets of question papers for each of the two class tests.
- **Class test duration:** 80 mins, in the class.
- **First test:** Sep 3 (Th). **Second test:** Nov 3 (T)

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- *Practice problems* will be provided in the slides (in the first half) and over Teams (in the second half).
- In a class test, a student will be asked to solve problems (which will be chosen from among the practice problems or slight variants of them).
- There will be multiple sets of question papers for each of the two class tests.
- The syllabus for the tests will be announced on the course homepage, over Teams, and in the class.

Mid-term and End-term exams

- Would be 3 hr long written tests.
- **When?** Exact dates will be announced on the course homepage, over Teams, and in the class.
- The problems given in these exams **need not** be from among the practice problems.

Mid-term and End-term exams

- **Mid-term syllabus:** Everything covered before the mid-term exam week.

- **End-term syllabus:** Everything covered *after the mid-term* and before the end-term exam.

Tutorial sessions?

- If required, we will have doubt-clearing sessions or tutorial sessions. The dates will be announced on the course homepage, over Teams, and in the class.

Introduction

What is an algorithm?

- An algorithm is a well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output in a *finite amount of time*.
- An algorithm is a tool for solving a well-specified (computational) problem.

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- An algorithm is a tool for solving a well-specified (computational) problem.
- Example. *(Sorting problem)* Sort a sequence of numbers into monotonically increasing order.

$$\underbrace{(31, 41, 59, 26, 41, 58)}_{\text{Input (instance)}} \mapsto \underbrace{(26, 31, 41, 41, 58, 59)}_{\text{Output}}$$

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- An algorithm is a tool for solving a well-specified (computational) problem.
- An algorithm solves a problem if for *every* input instance, it halts, and outputs the correct solution.
- An algorithm can be specified as a pseudocode or a computer program, or as a hardware design.

What is a pseudocode?

```
SUM-ARRAY(A, n)
1  sum = 0
2  for i = 1 to n
3      sum = sum + A[i]
4  return sum
```

- The above pseudocode (algorithm) takes an array A and a number n as input and returns the sum of the first n values of the array.

Problems and Applications

Problems	Applications
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Topological Sorting	To indicate precedence among events

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We'll discuss all these problems in later lectures.

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Linear Programming	Optimizing an objective, given limited resources and competing constraints
Encryption/Decryption	E-commerce

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- Algorithms to solve these and many other problems are at the *core of most technologies* used in contemporary computers.

What is a data structure?

- A data structure is a way to store and organize data in order to facilitate access and modifications.

- Using appropriate data structures is an important part of algorithm design.

- Examples of data structures. Arrays, lists, binary trees, hash tables, etc. We will study these in later lectures.

Techniques for Algorithm Design

- We'll also discuss the following broad techniques for designing and analysing algorithms:
  - Divide-and-Conquer
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  - Amortized Analysis
- These techniques are used to design time efficient algorithms. Question: Other than time, what other measure of efficiency can you think of? Memory space.

Insertion Sort:
Our first algorithm

Insertion Sort

- Sorting problem. Input is an array of n values $(a_1, \ldots, a_n)$. Output a reordering $(a'_1, \ldots, a'_n)$ of the input array such that $a'_1 \leq \ldots \leq a'_n$.

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- Insertion sort works the way you might sort a hand of playing cards.
- Idea. Remove one card from the pile and insert it into the correct position.

Insertion Sort: Pseudocode

```
INSERTION-SORT(A, n)
1  for i = 2 to n
2      key = A[i]
3      // Insert A[i] into the sorted subarray A[1 : i − 1].
4      j = i − 1
5      while j > 0 and A[j] > key
6          A[j + 1] = A[j]
7          j = j − 1
8      A[j + 1] = key
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- Input is an array A of values and the number n of values to sort.

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- Analyzing a pseudocode (algorithm) means proving its **correctness** and bounding the resources (which in our case would be **computational time**) that the algorithm requires.

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- Correctness. Inside the **for**-loop (*Steps 2-8*) we assume that the subarray $A[1 : i-1]$ is already sorted.

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- Correctness. Inside the **for**-loop (*Steps 2-8*) we assume that the subarray $A[1 : i-1]$ is already sorted. *Step 2* stores the value of $A[i]$ in key. This frees up the i^{th} position of A.

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6          A[j + 1] = A[j]
7          j = j − 1          // At this point A[j+1] is free
8      A[j + 1] = key
```

- Correctness. The **while**-loop (*Steps 5-7*) finds the correct position of key in $A[1 : i]$ by shifting values to the right. *Step 8* then inserts the key in the correct position–the $(j+1)^{th}$ position–once the loop exits.

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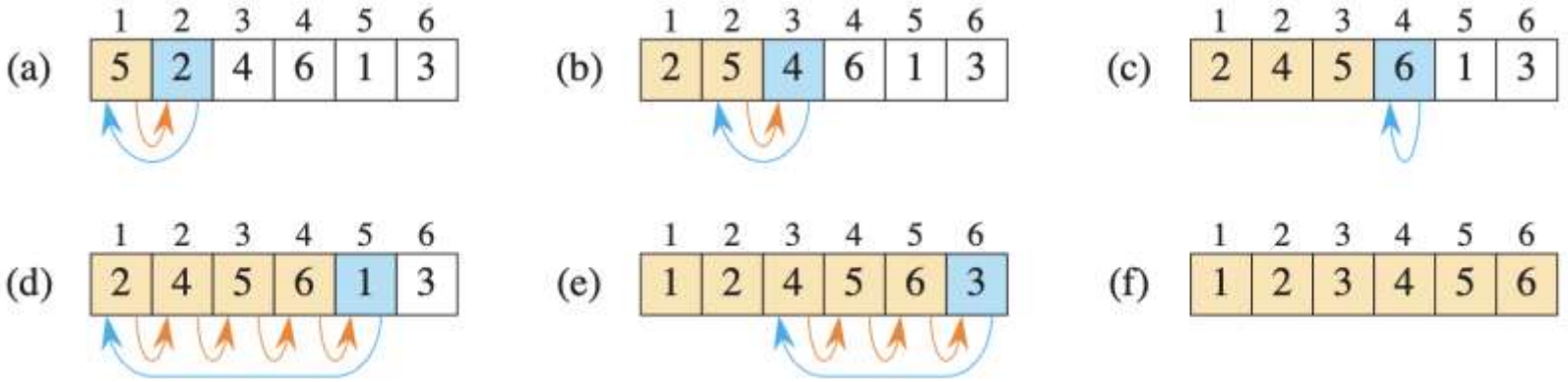

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- Before we discuss the computational/running time (a.k.a **time complexity**) of the algorithm, we need to specify the **computational model**.
- Our model should be reasonably realistic.

The RAM computational model

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- In the RAM model, instructions execute *sequentially* with no concurrent operations.
- Each instruction takes the *same amount of time* as any other instruction.
- Each data access (such as using the value of a variable or storing in a variable) takes the *same amount of time* as any other data access.

The RAM computational model

- We assume a one-processor random access memory (RAM) model of computation.
- The RAM model contains instructions *commonly found in real computers*: arithmetic (addition, subtraction, multiplication, division, etc.), data movement (load, store, copy), and control (conditional and unconditional branching, subroutine call and return).
- RAM model analyses are usually *excellent predictors of performance* on actual machines.

Running time or Time complexity

- We'll describe the running time of an algorithm *as a function of the size of its input*.
- Input size depends on the problem being studied:
  - For an array, size is the *number of items* in it.
  - If the input consists of two integers (as in the integer multiplication problem), then size means the *number of bits* needed to represent the input.

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  - If the input consists of two integers (as in the integer multiplication problem), then size means the *number of bits* needed to represent the input.
- The running time on a particular input is the number of instructions or data accesses executed.

Worst-case running time

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  - The worst-case running time *gives an upper bound* on the running time for any input.
  - For some algorithms, the *worst-case occurs fairly often*. For example, searching a database when the information is absent.
- We'll be interested in the rate of growth or the order of growth of the worst-case running time.

Insertion Sort: Running time

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1  for i = 2 to n
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- The worst-case running time of the **while**-loop (*Steps 5-7*) is at most $c \cdot i$, where c is a constant.
- So, the worst-case running time of the algorithm is at most $c' \cdot n^2$, where c' is a constant.

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- It's time to introduce the order notations…

Order Notations

Asymptotic growth of functions

- We'll be interested in the asymptotic efficiency of algorithms.
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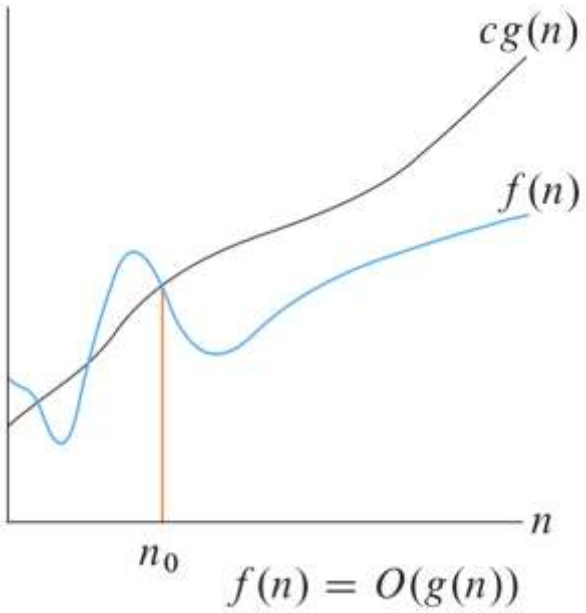
- That is, we'll be concerned with how the running time of an algorithm *grows with the input size*.

- We usually consider one algorithm to be asymptotically more efficient than another if its *worst-case running time has a lower order of growth*.
- Usually, an algorithm that is asymptotically more efficient is the best choice for all but small inputs.

Big-oh (O) notation

- O-notation describes an asymptotic upper bound.
- Definition. For a function $g(n)$,

$O(g(n)) = \{f(n)$: $\exists$ positive constants c and n_0 s.t. $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0\}$.



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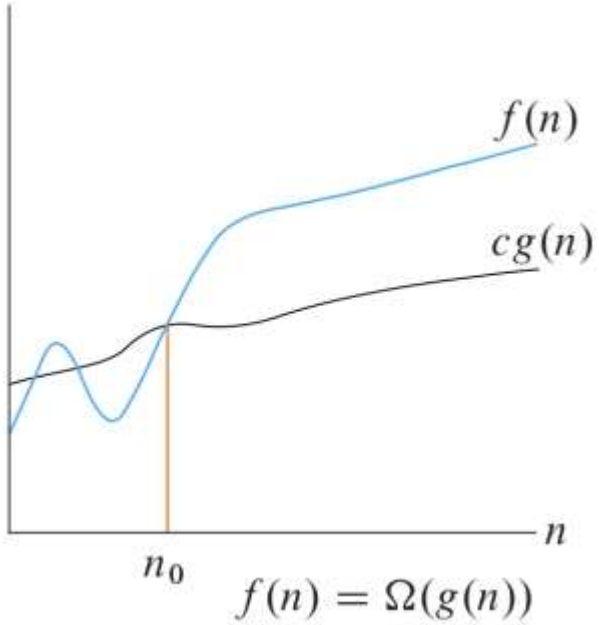
- We usually write "$f(n) = O(g(n))$" and say "$f(n)$ is big-oh of $g(n)$" to mean "$f(n) \in O(g(n))$".
- Example. $4n^2 + 100n + 500 = O(n^2)$. We use asymptotic notations to provide the *simplest bound*.

Big-omega (Ω) notation

- Ω-notation describes an asymptotic lower bound.
- Definition. For a function $g(n)$,

$\Omega(g(n)) = \{f(n)$: $\exists$ positive constants c and n_0 s.t. $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0\}$.



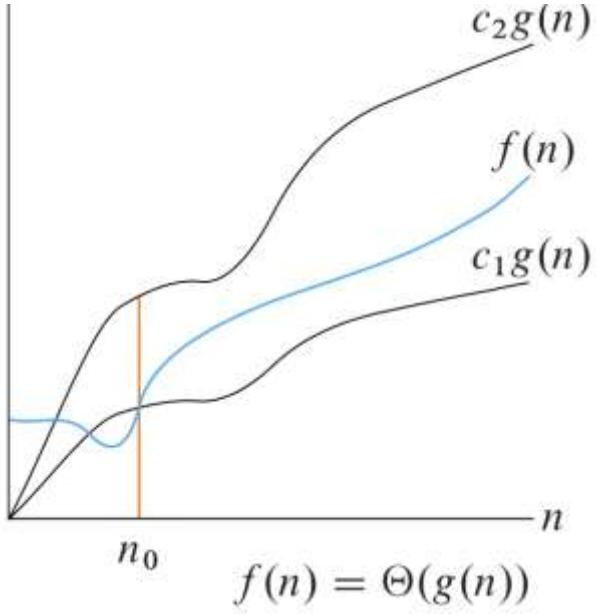
- Example. $2n^3 + 4n^2 + 100n + 500 = \Omega(n^3)$.

Theta (Θ) notation

- Θ-notation describes an asymptotically tight bound.
- Definition. For a function $g(n)$,

$\Theta(g(n)) = \{f(n)$: $\exists$ positive constants c_1, c_2 & n_0 s.t.

$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0\}$.



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$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0\}$.

- Obs. For any two functions $f(n)$ and $g(n)$, we have $f(n) = \Theta\big(g(n)\big)$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

Little-oh (o) notation

- o-notation denotes an upper bound that is not tight.
- Definition. For a function $g(n)$, $o(g(n))$ is the set

$\{f(n) :$ $\forall$ constant $c > 0$, $\exists$ a constant $n_0 > 0$ s.t.
$0 \leq f(n) < cg(n)$ for all $n \geq n_0\}$.

- Example. $20n = o(n^2)$, but $20n^2 \neq o(n^2)$.

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- The main difference between O and o is: In $f(n) = O(g(n))$, the condition $0 \leq f(n) \leq cg(n)$ holds *for some* constant $c > 0$, but in $f(n) = o(g(n))$, the condition $0 \leq f(n) < cg(n)$ holds *for all* $c > 0$.

Little-omega (ω) notation

- ω-notation denotes a lower bound that is not tight.
- Definition. For a function $g(n)$, $\omega(g(n))$ is the set

$\{f(n) :$ $\forall$ constant $c > 0$, $\exists$ a constant $n_0 > 0$ s.t.
$0 \leq cg(n) < f(n)$ for all $n \geq n_0\}$.

- Example. $\frac{n^2}{20} = \omega(n)$, but $\frac{n^2}{20} \neq \omega(n^2)$.

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- Example. $\frac{n^2}{20} = \omega(n)$, but $\frac{n^2}{20} \neq \omega(n^2)$.
- Obs. $f(n) = \omega(g(n))$ if and only if $g(n) = o(f(n))$.

Comparing functions

Transitivity:

$$
\begin{aligned}
f(n) &= \Theta(g(n)) \text{ and } g(n) = \Theta(h(n)) \text{ imply } f(n) = \Theta(h(n)) , \\
f(n) &= O(g(n)) \text{ and } g(n) = O(h(n)) \text{ imply } f(n) = O(h(n)) , \\
f(n) &= \Omega(g(n)) \text{ and } g(n) = \Omega(h(n)) \text{ imply } f(n) = \Omega(h(n)) , \\
f(n) &= o(g(n)) \text{ and } g(n) = o(h(n)) \text{ imply } f(n) = o(h(n)) , \\
f(n) &= \omega(g(n)) \text{ and } g(n) = \omega(h(n)) \text{ imply } f(n) = \omega(h(n)) .
\end{aligned}
$$

Reflexivity:

$$
\begin{aligned}
f(n) &= \Theta(f(n)) , \\
f(n) &= O(f(n)) , \\
f(n) &= \Omega(f(n)) .
\end{aligned}
$$

Symmetry:

$$f(n) = \Theta(g(n)) \text{ if and only if } g(n) = \Theta(f(n)) .$$

Comparing functions

Transpose symmetry:

$$f(n) = O(g(n)) \quad \text{if and only if} \quad g(n) = \Omega(f(n)),$$
$$f(n) = o(g(n)) \quad \text{if and only if} \quad g(n) = \omega(f(n)) .$$

Practice problems

Practice Problem 1

Consider the ***searching problem***:

Input: A sequence of n numbers $\langle a_1, a_2, \ldots, a_n \rangle$ stored in array $A[1:n]$ and a value x.

Output: An index i such that x equals $A[i]$ or the special value NIL if x does not appear in A.

Write pseudocode for ***linear search***, which scans through the array from beginning to end, looking for x. ~~Using a loop invariant, prove that your algorithm is~~

Practice Problem 2

Consider the problem of adding two n-bit binary integers a and b, stored in two n-element arrays $A[0:n-1]$ and $B[0:n-1]$, where each element is either 0 or 1, $a = \sum_{i=0}^{n-1} A[i] \cdot 2^i$, and $b = \sum_{i=0}^{n-1} B[i] \cdot 2^i$. The sum $c = a + b$ of the two integers should be stored in binary form in an $(n+1)$-element array $C[0:n]$, where $c = \sum_{i=0}^{n} C[i] \cdot 2^i$. Write a procedure ADD-BINARY-INTEGERS that takes as input arrays A and B, along with the length n, and returns array C holding the sum.

Practice Problem 3

Consider sorting n numbers stored in array $A[1:n]$ by first finding the smallest element of $A[1:n]$ and exchanging it with the element in $A[1]$. Then find the smallest element of $A[2:n]$, and exchange it with $A[2]$. Then find the smallest element of $A[3:n]$, and exchange it with $A[3]$. Continue in this manner for the first $n-1$ elements of A. Write pseudocode for this algorithm, which is known as ***selection sort***. ~~What loop invariant does this algorithm maintain? Why does it~~

- Analyze the worst-case running time of selection sort.

Practice Problem 4

Bubblesort is a popular, but inefficient, sorting algorithm. It works by repeatedly swapping adjacent elements that are out of order. The procedure BUBBLESORT sorts array $A[1:n]$.

```
BUBBLESORT(A, n)
1  for i = 1 to n − 1
2      for j = n downto i + 1
3          if A[j] < A[j − 1]
4              exchange A[j] with A[j − 1]
```

- Question. What is the worst-case running time of BUBBLESORT?

Practice Problem 5

- Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

- Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

- Show that $k \lg k = \Theta(n)$ implies $k = \Theta(n / \lg n)$.

Practice Problem 6

Let

$$p(n) = \sum_{i=0}^{d} a_i n^i ,$$

where $a_d > 0$, be a degree-d polynomial in n, and let k be a constant. Use the definitions of the asymptotic notations to prove the following properties.

a. If $k \geq d$, then $p(n) = O(n^k)$.

b. If $k \leq d$, then $p(n) = \Omega(n^k)$.

c. If $k = d$, then $p(n) = \Theta(n^k)$.

d. If $k > d$, then $p(n) = o(n^k)$.

e. If $k < d$, then $p(n) = \omega(n^k)$.

Practice Problem 7

Indicate, for each pair of expressions (A, B) in the table below whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Write your answer in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

$$\lg n = \log_2 n$$