Fast Actively Secure OT Extension for Short Secrets

A THESIS

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BY

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Declaration of Originality

I, **Ajith S**, with SR No. **11589** hereby declare that the material presented in the thesis titled

Fast Actively Secure OT Extension for Short Secrets

represents original work carried out by me in the **Department of Computer Science and Automation** at **Indian Institute of Science** during the years **2014-2017**.
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Advisor Name: Advisor Signature

DEDICATION

I dedicate my sincere efforts to my sweet and loving

Achan & Amma

whose love, affection and words of encouragement guided me in achieving success and honor,

Along with all my beloved

Teachers

for being a great source of inspiration

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Abstract

Oblivious Transfer (OT) is one of the most fundamental cryptographic primitives with widespread application in general secure multi-party computation (MPC) as well as in a number of tailored and special-purpose problems of interest such as private set intersection (PSI), private information retrieval (PIR), contract signing to name a few. Often the instantiations of OT require prohibitive communication and computation complexity. OT extension protocols are introduced to compute a very large number of OTs referred as *extended* OTs at the cost of a small number of OTs referred as *seed* OTs.

We present a fast OT extension protocol for small secrets in active setting. Our protocol when used to produce 1-out-of-n OTs outperforms all the known actively secure OT extensions. Our protocol is built on the semi-honest secure extension protocol of Kolesnikov and Kumaresan of CRYPTO'13 (referred as KK13 protocol henceforth) which is the best known OT extension for short secrets. At the heart of our protocol lies an efficient consistency checking mechanism that relies on the linearity of Walsh-Hadamard (WH) codes. Asymptotically, our protocol adds a communication overhead of $\mathcal{O}(\mu \log \kappa)$ bits over KK13 protocol irrespective of the number of extended OTs, where κ and μ refer to computational and statistical security parameter respectively. Concretely, our protocol when used to generate a large enough number of OTs adds only 0.011-0.028% communication overhead and 4-6% runtime overhead both in LAN and WAN over KK13 extension. The runtime overheads drop below 2% when in addition the number of inputs of the sender in the extended OTs is large enough.

As an application of our proposed extension protocol, we show that it can be used to obtain the most efficient PSI protocol secure against a malicious receiver and a semi-honest sender.

Publications based on this Thesis

- Arpita Patra, Pratik Sarkar and **Ajith Suresh**. *Fast Actively Secure OT Extension for Short Secrets*. In The Network and Distributed System Security Symposium (NDSS) 2017

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Chapter 1

Introduction

The word cryptography means **secret** (crypto-) **writing** (-graphy). Traditionally, cryptography deals with study of techniques for secure communication. The problem of secure communication is formulated to enable secure transfer of sensitive and private information. In a secure communication problem, a sender wants to send its private information to a receiver in a way that no third party gets the information by eavesdropping over the communication medium. Though it is non-trivial to solve the problem of secure communication, the goal is well-stated and well-understood. We know how to solve the problem, namely with the help of *encryption schemes*. A more pressing and challenging problem is achieving both privacy and computation on private data simultaneously. Following are a few examples that demand protecting privacy of data while they undergo computation:

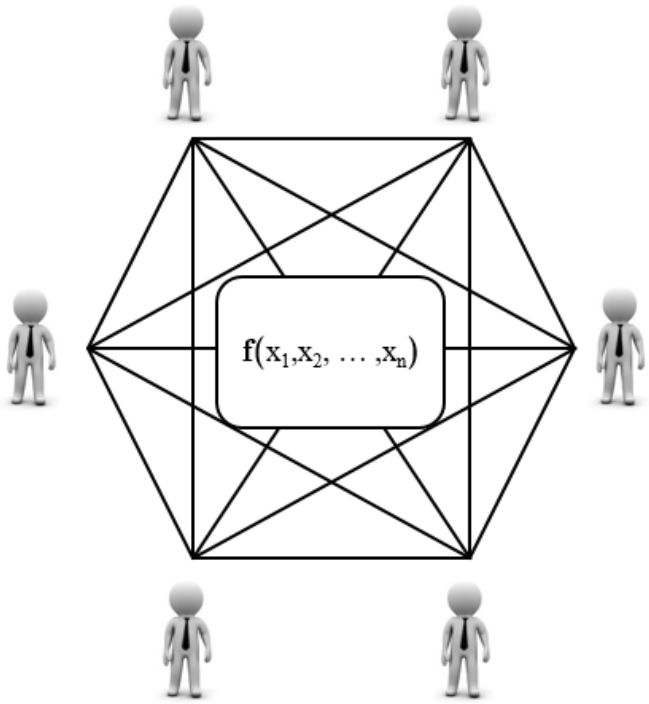
- **Secure Auction:** Every party possesses bid amount as its private input. The goal is to compute the maximum bid and the corresponding bidder ensuring nothing beyond the winner and the winning bid are revealed.
- **Satellite Collision:** The orbital information of satellite is private data possessed by a country which owns it. Different satellites may have orbits in close proximity and collision among such satellites may lead to extensive damage. In order to prevent such collisions, we need to compute the collision probability without leaking the highly accurate positional information of the satellites.
- **Privacy Preserving Data Mining:** Hospitals possesses patient records, which are deemed private. These records may require computation. For instance, several leading hospitals in India would like to find the number of patients who have been infected with a particular disease between say, 2015 and 2017 without revealing their patient database

to each other.

There are several other scenarios like above that demand both privacy and computation on private data simultaneously such as secure benchmarking, secure set intersection, private information retrieval etc. Here classical cryptography fails. Fortunately, the area of Secure Computation or more commonly known as *Secure Multi-party Computation* (MPC) has evolved naturally to tackle the above problem. In this thesis, we focus on the most fundamental building block of MPC, known as Oblivious Transfer (OT) [NP05, Kil88, BCR86, EGL85, Rab81].

1.1 Secure multi-party computation

MPC [GMW87, DO10, BDOZ11, DPSZ12, AJL+12, GGHR14, LPSY15, BHP17, ACJ17, BGW88, RB89, BMR90, BB89, Bea91, DN07, BFO12, BH06, BH07, BH08, CDD+99, CHP13, Yao82, LP07, RFZ+13, NO09, JS07, MF06, Woo07, LP12, IPS08, AMPR15, SS13, FJN+13, Lin13, HKE13, HKK+14, RR16, MR17], introduced by Andew Chi Chih Yao [Yao82], is arguably regarded as the most fundamental problem in cryptography. Being a powerful abstraction, it can model any cryptographic task. The problem is defined as follows: We have a set of n distrusting parties $\{P_1, \ldots, P_n\}$, each with its own private input $x_1, \ldots, x_n$. They want to compute some publicly known function f on their inputs without disclosing their inputs. The distrust among



the parties is formalized by having an *adversary* that may corrupt some of the parties. We consider a *monolithic or centralized* adversary i.e., if two or more parties are corrupted, we assume that they collude with each other. The parties under the control of the adversary are called "corrupt" and the remaining parties are called "honest". Informally, the basic properties that an MPC protocol aims to achieve include:

- **Correctness:** The honest parties output the correct function value.
- **Privacy:** The adversary does not learn anything that cannot be efficiently derived from the inputs and outputs of the corrupted parties.
- **Independence of Inputs:** Corrupted parties must choose their inputs independently of the honest parties' inputs.

The approach used by a generic secure computation protocol is to "securely" evaluate Boolean circuit (with AND, OR, XOR gates) or arithmetic circuit (with $+, \times$ gates) representing the function f to be computed. By "secure circuit evaluation" we mean that the circuit will be evaluated in such a way that nothing other than the circuit output that represents the function output must be revealed during the circuit evaluation. The works on MPC mostly follow either of the two paradigms: Yao's garbled circuit based protocol [Yao86] or the GMW protocol [GMW87]. Both of these protocols require the given function to be represented by a Boolean circuit. While GMW demands interaction for each AND gate of the circuit causing an overall round complexity in the order of the multiplicative depth of the circuit, Yao's protocol runs in constant rounds. Both these approaches rely on the most important building block of MPC, namely OT. While GMW approach requires one instance of OT per AND gate, Yao-style protocols require one OT per input bit. Therefore, OT is a fundamental and extremely useful tool for building MPC protocols.

1.2 Oblivious Transfer

OT is perhaps the most fundamental primitive in cryptographic protocol theory. It is a two party protocol between a sender S and a receiver R. The sender holds an array of inputs and the receiver holds an index indicating its intended pick from the sender's array. OT allows the sender to send the receiver's selected input while preserving the secrecy of the sender's other inputs on the one hand and the choice of the receiver on the other.

The necessity and sufficiency of OT for MPC [Kil88, GV87, GMW87, Yao86] backs the theoretical importance of OT. On the practical front, OT has been pivotal in building several state-of-the-art practically efficient general MPC protocols [Lin16, LP15, LR15, HKK+14,

FJN+13, SS13, NO09] and several protocols for special-purpose problems of interest such as private set intersection (PSI) [PSSZ15, PSZ14, DCW13]. There is a fundamental limitation to OT's efficiency as it is unlikely that OT is possible without public-key cryptography and solely relying on symmetric-key cryptography [IR89]. The OT extension protocols [KOS15, ALSZ15, KK13, ALSZ13, NNOB12, IKNP03, Bea96] have been introduced to theoretically circumvent the above limitation of OTs. They produce a large number of OTs referred as *extended* OTs from a small number of OTs referred as *seed* OTs and symmetric-key primitives. When the goal is to generate a large number of OTs which is usually the case for the applications of OT, the amortized cost of generating a single OT via OT extensions turns out to be a constant number of symmetric-key operations. So most of the known practically efficient general and special-purpose MPC protocols are byproduct of concretely efficient OT extension protocols.

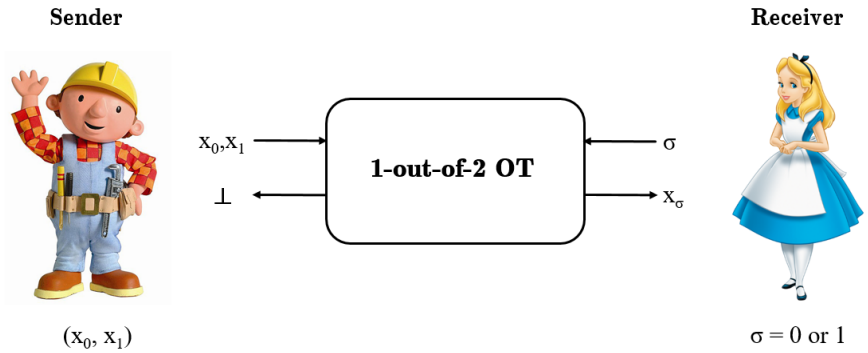


Figure 1.1: A 1-out-of-2 Oblivious Transfer

Of particular interest to cryptographic community are the following variants of OT: (a) In a 1-out-of-2 OT [EGL85], S holds two inputs x_0, x_1, and R holds a *choice bit* b. The output to R is x_b and no other party learns anything. (b) A straight-forward extension of 1-out-of-2 OT is 1-out-of-n OT [BCR86] where S holds n inputs and R holds a choice index of $\log n$ bits. While the first kind finds application in MPC [GMW87, Yao82], the second kind is useful in PSI [PSSZ15, PSZ14], symmetric PIR [NP05], and oblivious sampling [NP05], oblivious polynomial evaluation [NP99]. As discussed below, attempts have been made to construct OT extension protocols to output both the above kinds of OTs.

1.3 OT Extensions

The theoretical feasibility of OT extension was proved by Beaver [Bea96]. Ishai, Kilian, Nissim and Petrank [IKNP03] (referred as IKNP protocol henceforth) presented the first efficient OT

extension protocol that builds on κ seed OTs and requires computing and sending just two hash values per extended OT. In [ALSZ13], IKNP protocol has seen several optimizations that boost both its communication and computation complexity. Specifically, the communication per extended OT is brought down to one hash value for a special case where the extended OTs are needed for random inputs of the sender. The computation bottleneck for implementing matrix transposition is tackled by introducing a cache-oblivious algorithm. Yet another contribution from [ALSZ13] is their crucial observation that the actual bottleneck in the runtime of IKNP protocol results from its communication time, particularly in wide area networks (WANs) that have high latency and low bandwidth. In a first of its kind approach, Kolesnikov and Kumaresan [KK13] (referred as KK13 protocol henceforth) presented an OT extension protocol that outputs 1-out-of-n OTs starting from 2κ 1-out-of-2 seed OTs and relying on specifics of Walsh-Hadamard (WH) codes. KK13 protocol improves over all its predecessors (including IKNP) customized to generate 1-out-of-n OTs by a factor $\mathcal{O}(\log n)$ in communication complexity when the inputs of the extended OTs are of short size. So far KK13 protocol remains to be the most efficient way of generating 1-out-of-n OTs for short inputs. All the protocols discussed above work when the adversary is assumed to be *semi-honest*. A passive or semi-honest adversary follows the protocol specification but attempts to learn more than allowed by inspecting the protocol transcript. An adversary is referred as *active* or *malicious* when it behaves in any arbitrary way in an attempt to break the security of the protocol.

OT extension literature finds numerous attempts to achieve active security. All the attempts restrict their attention in transforming the semi-honest secure IKNP protocol to an actively secure one. Since the IKNP protocol is resilient to any malicious behavior of the sender, an actively secure IKNP style protocol needs to enforce honest behaviour for the receiver. Adding consistency checks for the receiver has been the strategy followed in all the known constructions. The efficiency (both communication and computation wise) of the consistency checks defines the overhead for an actively secure IKNP style protocol. The consistency check introduced in [IKNP03] employs expensive cut-and-choose technique on μ parallel instances of the semi-honest IKNP protocol where μ is a statistical security parameter. [HIKN08, Nie07] proposes consistency checks per extended OTs. This is improved in [NNOB12] where the checks are done per seed OT. In order to tackle information leakage in their consistency check, [NNOB12] needs to start with $\frac{8}{3}\kappa$ seed OTs which is $\frac{8}{3}$ times more than what IKNP protocol needs. This inflates their concrete communication and computation complexity by the same factor. [ALSZ15] improves over [NNOB12] by trading computation in consistency checks for a reduced number of seed OTs. Namely, the OT extension of [ALSZ15] requires $\kappa + 1.55\mu$ seed OTs compared to $\frac{8}{3}\kappa$ of [NNOB12] and thus improves the communication done via seed OTs. In a

recent work, [KOS15] reports the most efficient actively secure IKNP style protocol that brings back the number of seed OTs to κ and handles the information leakage in the consistency check by sacrificing $\kappa+\mu$ extended OTs. The check requires an $\mathcal{O}(\kappa)$ bits communication irrespective of the number of extended OTs and two finite field operations per extended OT.

Above we concentrated on practically efficient OT extension literature. Some interesting theoretical questions on OT extension are addressed in [Lar14, LZ13].

1.4 Our Contribution

We present an actively secure OT extension for short secrets building upon the semi-honest secure protocol of [KK13]. Like KK13 protocol, our extension protocol turns 1-out-of-2 seed OTs to 1-out-of-n extended OTs. Similar to IKNP protocol, KK13 protocol is secure against any malicious behaviour of sender but falls apart in the face of a maliciously corrupt receiver. We present a concrete attack on KK13 and add an efficient consistency check to enforce correct behaviour of the receiver. Our check relies on the linearity of WH codes. Combined with an additional trick, our efficient consistency check incurs a communication of $\mathcal{O}(\mu \log \kappa)$ bits irrespective of the number of generated extended OTs. Asymptotically, our OT extension matches the KK13 protocol in every respect. Table 1.1 shows the efficiency of various OT extension protocols achieving $2^{-\kappa}$ computational security and $2^{-\mu}$ statistical security for producing m 1-out-of-n OTs with ℓ-bit inputs of the sender. The following parameters have been used for comparison: (i) number of seed OTs, (ii) communication complexity and (iii) computation complexity in terms of number of hash value computations.

Table 1.1: Asymptotic cost of various OT extensions for producing m 1-out-of-n OTs with ℓ-bit inputs of the sender and for achieving $2^{-\kappa}$ computational security and $2^{-\mu}$ statistical security.

Reference	# Seed OTs	Communication (bits) / Computation (# hashes)	Security
[KK13]	2κ	$\mathcal{O}(m(\kappa + n\ell))$	semi-honest
[IKNP03]	κ	$\mathcal{O}(m(\kappa \log n + n\ell))$	semi-honest
[NNOB12]	$\frac{8}{3}\kappa$	$\mathcal{O}(m(\kappa \log n + n\ell))$	active
[ALSZ15]	$\kappa + 1.55\mu$	$\mathcal{O}(m(\kappa \log n + n\ell))$	active
[KOS15]	κ	$\mathcal{O}(m(\kappa \log n + n\ell))$	active
This Paper	2κ	$\mathcal{O}(m(\kappa + n\ell))$	active

Concretely, our protocol when used to generate large enough number of OTs adds only 0.011-0.028% communication overhead and 4-6% runtime overhead both in LAN and WAN over KK13 protocol. The runtime overheads drop below 2% when in addition the number of

inputs of the sender in the extended OTs is large enough. Our construction put in the context of other OT extensions are presented in Table 1.2. The table presents figures for generating 1.25×10^6 1-out-of-16 OTs with sender's input length as 4 bits. The overheads are calculated with respect to KK13 protocol. The implementation of [KOS15] is not available in the same platform as the other OT extensions given in the table. As per the claim made in [KOS15], the runtime of their OT extension bears an overhead of 5% with respect to IKNP protocol both in LAN and WAN. So the runtime and overhead in runtime of [KOS15] with respect to KK13 protocol are calculated based on that claim. As evident from Table 1.2, our protocol when used to compute 1-out-of-n OTs with short inputs of the sender outperforms all the known actively secure OT extensions and secures the second best spot among all the OT extension protocols listed in Table 1.2 closely trailing KK13 which is the overall winner. More elaborate empirical results supporting the above claim with varied number of extended OTs and varied number of inputs of the sender in the extended OTs appear later in the paper.

Table 1.2: Concrete cost of various OT extension protocols for producing 1.25×10^6 1-out-of-16 OTs with sender's input length as 4 and for achieving computational security of 2^{-128} and statistical security of 2^{-40}.

Reference	# Seed OTs	Comm (in MB)	Runtime (in sec)		Overhead w.r.t. [KK13] (in %)		
			LAN	WAN	Communication	Runtime in LAN	Runtime in WAN
[KK13]	256	47.69	21.68	115.34	0	0	0
[IKNP03]	128	87.74	24.07	133.81	84	11.02	16
[NNOB12]	342	215.95	24.84	143.20	352.7	14.6	24.14
[ALSZ15]	190	166.54	24.81	158.6	249.1	14.4	37.5
[KOS15]	128	> 87.74	> 25.27	> 140.5	> 84	> 16.5	> 21.8
This Paper	256	47.70	22.50	121.94	**0.028**	**3.78**	**5.72**

Lastly, the OT extensions presented in all the works in the table except [KK13] inherently produce 1-out-of-2 OTs. The transformation from 1-out-of-2 to 1-out-of-n OT given in [NP05] is used to transform their extended OTs to 1-out-of-n OTs. The transformation that works for reverse direction [NP05] is unfortunately *not* maliciously secure. This prevents us from claiming a similar gain when our protocol is used to generate 1-out-of-2 OTs. We leave open the question of finding an efficient actively secure transformation from 1-out-of-n to 1-out-of-2 OT.

We show an interesting application of our proposed extension protocol in OT-based PSI protocols. Specifically, we use our maliciously secure OT extension in the PSI protocol of [Lam16] to obtain the most efficient PSI protocol that is maliciously secure against a corrupt receiver and semi-honestly secure against a corrupt sender. In brief, a PSI protocol between two parties, namely a sender S and a receiver R holding sets $X = \{x_1, x_2, \ldots x_{n_1}\}$ and $Y = \{y_1, y_2, \ldots y_{n_2}\}$ respectively, outputs the intersection $X \cap Y$ to the receiver and nothing to the

sender. As evident from the theoretical and experimental results presented in this work, our maliciously secure OT extension protocol is a better choice compared to the existing maliciously secure extension protocols [ALSZ15, NNOB12, KOS15] when 1-out-of-n OTs are required as output. As PSI employs 1-out-of-n (instead of 1-out-of-2) OTs, our extension protocol fits the bill. Lastly, we find a concrete vulnerability for the malicious corrupt receiver case in Lambæk's PSI protocol when semi-honest KK13 OT protocol is used in it. This confirms Lambæk's concern of privacy breach of his PSI protocol that may result from privacy breach of the underlying OT protocols and further confirms the necessity of maliciously secure OT extension in Lambæk's PSI protocol.

1.5 Outline of this Thesis

This thesis starts by introducing the basics of MPC and a high-level overview of the preliminaries relevant to our work. This is followed by the problem formulation and our proposed solutions. We now present the thesis outline.

- **Chapter 2:** This chapter provides an overview of preliminary topics such as Walsh-Hadamard (WH) Codes, Universal Composability (UC) Model and the security notions.
- **Chapter 3:** In this chapter, we describe the efficient protocol of KK13 on which our actively secure protocol is built upon. We then present a concrete attack on KK13 when the receiver is considered to be maliciously secure.
- **Chapter 4:** In this chapter, we first give a technical overview of our protocol. We then move onto a formal description of our protocol, followed by a rigorous security proof. Next, we present our empirical findings and analyze the efficiency of our protocol in comparison to the existing state-of-the-art protocols. Finally, we describe an interesting application of our proposed protocol in OT-based Private Set Intersection (PSI) protocols.
- **Chapter 5:** We conclude with summary of the thesis and possible future directions to this work.

Chapter 2

Preliminaries

2.1 Notations

We use $\oplus$ to denote bitwise XOR operation and $\odot$ to denote bitwise AND operation. We denote vectors in bold smalls and matrices in bold capitals. For a matrix $\mathbf{A}$, we let $\mathbf{a}_j$ denote the jth row of $\mathbf{A}$, and $\mathbf{a}^i$ denote the ith column of $\mathbf{A}$. For a vector $\mathbf{a}$, a_i denotes the ith element in the vector. For two vectors $\mathbf{a}$ and $\mathbf{b}$ of length p, we use the notation $\mathbf{a} \oplus \mathbf{b}$ to denote the vector $(a_1 \oplus b_1, \cdots, a_p \oplus b_p)$ and the notation $\mathbf{a} \odot \mathbf{b}$ to denote the vector $(a_1 \odot b_1, \cdots, a_p \odot b_p)$. The notation $\bigoplus_{j=1}^{m} \mathbf{a}_j$ denotes the XOR of m vectors, i.e. $\mathbf{a}_1 \oplus \cdots \oplus \mathbf{a}_m$. We denote by $\mathbf{a} \otimes \mathbf{b}$ the inner-product value $\bigoplus_{i=1}^{p} a_i \odot b_i$. Finally, suppose $c \in \{0,1\}$, then $c \odot \mathbf{a}$ denotes the vector $(c \odot a_1, \cdots, c \odot a_p)$. We denote by $a \leftarrow_R A$ the random sampling of a from a distribution A. We denote by $[x]$, the set of elements $\{1, \ldots, x\}$.

We denote by $\mathtt{HDI}$ a function that takes two binary vectors of same length and returns the indices where the input vectors are different. For a vector $\mathbf{c}$ of length, say p, and an index set $\mathcal{I} \subset [p]$, $\mathtt{PRN}_{\mathcal{I}}(\mathbf{c})$ denotes the pruned vector that remains after removing the bits of $\mathbf{c}$ corresponding to the indices listed in $\mathcal{I}$. For a set $\mathcal{C}$ of vectors $\{\mathbf{c}_1, \ldots, \mathbf{c}_m\}$, $\mathtt{PRN}_{\mathcal{I}}(\mathcal{C})$ denotes the set of pruned vectors $\{\mathtt{PRN}_{\mathcal{I}}(\mathbf{c}_1), \ldots, \mathtt{PRN}_{\mathcal{I}}(\mathbf{c}_m)\}$.

Security Parameters. We denote the statistical security parameter by μ and the cryptographic security parameter by κ. A negligible function in κ (μ) is denoted by $\mathtt{negl}(\kappa)$ ($\mathtt{negl}(\mu)$), while $\mathtt{negl}(\kappa, \mu)$ denotes a function which is negligible in both κ and μ. A function $\mathtt{negl}(\cdot)$ is *negligible* if for every polynomial $p(\cdot)$ there exists a value N such that for all $n > N$ it holds that $\mathtt{negl}(n) < \frac{1}{p(n)}$. We write PPT for probabilistic polynomial-time.

Oblivious Transfers. For oblivious transfers, we denote the sender by S and the receiver by R. In a 1-out-of-2 OT on ℓ bit strings, S holds two inputs x_0, x_1, each from $\{0,1\}^\ell$ and R holds a

choice bit b. The output to R is x_b and no other party learns anything. We denote a 1-out-of-2 OT on ℓ bit strings as $\binom{2}{1}$-OT_ℓ. We denote a 1-out-of-n OT on ℓ bit strings as $\binom{n}{1}$-OT_ℓ. Finally, we write $\binom{n}{1}$-OT_ℓ^m to denote m instances of $\binom{n}{1}$-OT_ℓ. Similarly, $\binom{2}{1}$-OT_ℓ^m denotes m instances of $\binom{2}{1}$-OT_ℓ.

2.2 Walsh-Hadamard (WH) Codes

Walsh-Hadamard (WH) code is a linear code over a binary alphabet $\mathbb{F}_2$ that maps messages of length p to codewords of length 2^p. We use WH code that maps messages of length $\log \kappa$ to codewords of length κ. For $\mathbf{x} \in \{0,1\}^{\log \kappa}$, $\mathtt{WH}(\mathbf{x})$ denotes the WH encoding of $\mathbf{x}$ defined as $\mathtt{WH}(\mathbf{x}) := (\mathbf{x} \otimes \mathbf{a})_{\mathbf{a} \in \{0,1\}^{\log \kappa}}$. It is the κ-bit string consisting of inner products of each $\log \kappa$-bit string $\mathbf{a}$ with $\mathbf{x}$. For each κ, the WH code, denoted by $\mathcal{C}^\kappa_{\mathtt{WH}}$ is defined as the set $\{\mathtt{WH}(\mathbf{x})\}_{\mathbf{x} \in \{0,1\}^{\log \kappa}}$. Note that $\mathcal{C}^\kappa_{\mathtt{WH}}$ contains κ codewords each of length κ bits. Our OT extension protocol relies on the following well-known property of WH codes.

Theorem 2.2.1 *The distance of $\mathcal{C}^\kappa_{\mathtt{WH}}$ is $\frac{\kappa}{2}$ when κ is a power of 2.*

2.3 Hash Function and Random Oracle Model

We use a hash function $H : \{0,1\}^* \to \{0,1\}^{\mathsf{Poly}(\kappa)}$ which we model as a random oracle. Namely, we prove the security of our protocol assuming that H implements a functionality $\mathcal{F}_{\mathrm{RAND}}$ which for different inputs x, returns uniform random output values from the range of $H(x)$.

2.4 Security Model

We consider two types of adversary: *semi-honest/passive* and *malicious/active*. Semi-honest is a naive adversarial model in which the adversary learns the entire internal information of the corrupted parties, but the corrupted parties do not deviate from the protocol specification. Security against this type of adversary ensures that inadvertent leakage of information between parties does not occur. In the malicious model, the adversary takes full control of the corrupted parties and can make them deviate arbitrarily from the protocol. Protocols that are secure against malicious adversaries provide high security guarantee.

We consider adversaries that are polynomially bounded. Such adversaries run in probabilistic polynomial-time (PPT). This captures the notion of feasible computation; any attack that cannot be carried out in polynomial-time is not considered a threat in real life. Security in this model is referred to as computational or cryptographic security and usually relies on hard number-theoretic problems.

We assume that the parties are connected by secure channels. Such channels can be emulated using secure encryption schemes.

2.4.1 The Universal Composability (UC) Security Model

We prove security of our protocol in the standard Universal Composability (UC) framework of Canetti [Can01]. The UC framework introduces a PPT environment $\mathcal{Z}$ that is invoked on the security parameter κ and an auxiliary input z and oversees the execution of a protocol in one of the two worlds. The "ideal" world execution involves dummy parties P_0 and P_1, an ideal adversary $\mathcal{S}$ who may corrupt one of the dummy parties, and a functionality $\mathcal{F}$. The "real" world execution involves the PPT parties P_0 and P_1 and a real world adversary $\mathcal{A}$ who may corrupt one of the parties. The environment $\mathcal{Z}$ chooses the input of the parties and may interact with the ideal/real adversary during the execution. At the end of the execution, it has to decide upon and output whether a real or an ideal world execution has taken place.

We let $\text{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}(1^\kappa, z)$ denote the random variable describing the output of the environment $\mathcal{Z}$ after interacting with the ideal execution with adversary $\mathcal{S}$, the functionality $\mathcal{F}$, on the security parameter 1^κ and z. Let $\text{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$ denote the ensemble $\{\text{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}}(1^\kappa, z)\}_{\kappa\in\mathbb{N},z\in\{0,1\}^*}$. Similarly let $\text{REAL}_{\Pi,\mathcal{A},\mathcal{Z}}(1^\kappa, z)$ denote the random variable describing the output of the environment $\mathcal{Z}$ after interacting in a real execution of a protocol Π with adversary $\mathcal{A}$, the parties, on the security parameter 1^κ and z. Let $\text{REAL}_{\Pi,\mathcal{A},\mathcal{Z}}$ denote the ensemble $\{\text{REAL}_{\Pi,\mathcal{A},\mathcal{Z}}(1^\kappa, z)\}_{\kappa\in\mathbb{N},z\in\{0,1\}^*}$.

Definition 2.4.1 *For $n \in \mathbb{N}$, let $\mathcal{F}$ be a functionality and let Π be an 2-party protocol. We say that Π securely realizes $\mathcal{F}$ if for every PPT real world adversary $\mathcal{A}$, there exists a PPT ideal world adversary $\mathcal{S}$, corrupting the same parties, such that the following two distributions are computationally indistinguishable:*

$$\text{IDEAL}_{\mathcal{F},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \text{REAL}_{\Pi,\mathcal{A},\mathcal{Z}}.$$

The $\mathcal{F}$-hybrid model. In order to construct some of our protocols, we use secure two-party protocols as subprotocols. The standard way of doing this is to work in a "*hybrid model*" where both the parties interact with each other (as in the real model) in the outer protocol and use ideal functionality calls (as in the ideal model) for the subprotocols. Specifically, when constructing a protocol Π that uses a subprotocol for securely computing some functionality $\mathcal{F}$, the parties run Π and use "ideal calls" to $\mathcal{F}$ (instead of running the subprotocols implementing $\mathcal{F}$). The execution of Π that invokes $\mathcal{F}$ every time it requires to execute the subprotocol implementing $\mathcal{F}$ is called the *$\mathcal{F}$-hybrid execution of* Π and is denoted as $\Pi^{\mathcal{F}}$. The hybrid ensemble $\mathbf{HYB}_{\Pi^{\mathcal{F}},\mathcal{A},\mathcal{Z}}(1^\kappa, z)$ describes $\mathcal{Z}$'s output after interacting with $\mathcal{A}$ and the parties P_0, P_1 running protocol $\Pi^{\mathcal{F}}$. By

UC definition, the hybrid ensemble should be indistinguishable from the real ensemble with respect to protocol Π where the calls to $\mathcal{F}$ are instantiated with a realization of $\mathcal{F}$.

Chapter 3

An Attack on [KK13] Protocol

The KK13 OT extension protocol is known to provide the best communication complexity among the existing constructions when the input length of the sender is 'short'. The protocol is proven to be secure against a semi-honest receiver and a malicious sender. It was *not* known if the protocol is secure against malicious receiver. We show that the protocol is *insecure* against a malicious receiver. We prove this by giving an attack that can be mounted by a maliciously corrupt receiver to break the security of the sender. Our finding sets the stage for a maliciously secure OT extension in KK13 style which is the concern of this paper. Below we recall the KK13 OT extension protocol prior to presenting our attack. We also briefly recall its efficiency analysis from [KK13].

3.1 KK13 OT Extension Protocol

The KK13 OT extension protocol constructs a $\binom{n}{1}$-OT_{ℓ}^{m} relying on an instance of $\binom{2}{1}$-$\mathsf{OT}_{\kappa}^{\kappa}$. We recall the simpler version of the protocol that reduces $\binom{n}{1}$-OT_{ℓ}^{m} to $\binom{2}{1}$-OT_{m}^{κ}. It is well-known that $\binom{2}{1}$-OT_{m}^{κ} can be constructed from $\binom{2}{1}$-$\mathsf{OT}_{\kappa}^{\kappa}$ with some additional cost.

Following the footstep of [IKNP03], KK13 OT extension allows the receiver to send an $m \times \kappa$ matrix column-wise to the sender using an instance of $\binom{2}{1}$-OT_{m}^{κ} where the sender acts as the receiver and vice versa. In [IKNP03] OT extension, the ith row of the transferred matrix allows the sender to create two pads for the two messages in the ith extended OT. One of the two pads is a function of the sender's input bit vector to $\binom{2}{1}$-OT_{m}^{κ} and thus is unknown to the receiver. The other pad is completely known to the receiver. The pad known to the receiver is used as the mask for the intended message of the receiver. The above allows the receiver to unmask and learn its intended message for each extended OT but nothing more. Going along the same line, KK13 OT extension allows the sender to create n pads for the n messages in the ith extended

OT using the ith row of the transferred matrix. Much like IKNP, the receiver knows exactly one pad out of the n pads and the pad it knows is in fact the mask for its intended message. All the remaining $n-1$ pads are function of the sender's input bit vector to $\binom{2}{1}$-OT_m^κ and thus are unknown to the receiver. The ability to generate n masks instead of just 2 from each of the rows of the transferred matrix is achieved by cleverly incorporating WH codewords from $\mathcal{C}_{\mathtt{WH}}^\kappa$ in each of the rows of the transferred $m \times \kappa$ matrix. The use of $\mathcal{C}_{\mathtt{WH}}^\kappa$ restricts the value of n to be at most κ.

The protocol uses WH code $\mathcal{C}_{\mathtt{WH}}^\kappa$ that consists of κ codewords each of length κ denoted as $(\mathbf{c}_1, \cdots, \mathbf{c}_\kappa)$. The receiver R chooses two random $m \times \kappa$ matrices $\mathbf{B}$ and $\mathbf{D}$ such that ith row of matrix $\mathbf{E} = \mathbf{B} \oplus \mathbf{D}$ is $\mathbf{c}_{r_i}$ where r_i is the input of the receiver for the ith extended OT. On the other hand, the sender S picks a κ bit length vector $\mathbf{s}$ uniformly at random. The parties then interact via $\binom{2}{1}$-OT_m^κ reversing their roles. Namely, the sender S acts as the receiver with input $\mathbf{s}$ and the receiver R acts as a sender with inputs $\{\mathbf{b}^j, \mathbf{d}^j\}_{j\in[\kappa]}$. After the execution of $\binom{2}{1}$-OT_m^κ, the sender holds an $m \times \kappa$ matrix $\mathbf{A}$ such that the ith row of $\mathbf{A}$ is the ith row of $\mathbf{B}$ xored with the bitwise AND of $\mathbf{s}$ and $\mathbf{c}_{r_i}$, i.e. $\mathbf{a}_i = (\mathbf{b}_i \oplus (\mathbf{s} \odot \mathbf{c}_{r_i}))$. With the ith row $\mathbf{a}_i$ of the matrix $\mathbf{A}$, the sender creates n pads for the n messages in the ith extended OT as follows: $\big\{H\big(i, \mathbf{a}_i \oplus (\mathbf{s} \odot \mathbf{c}_j)\big)\big\}_{j\in[n]}$ where H is a random oracle. The jth pad will be used to blind the jth message of the sender in the ith extended OT. It is easy to note that the pad for the r_ith message is $H(i, \mathbf{b}_i)$ (since $\mathbf{a}_i = (\mathbf{b}_i \oplus (\mathbf{s} \odot \mathbf{c}_{r_i}))$) which the receiver can compute with the knowledge of $\mathbf{B}$ matrix. For the jth message where j is different from r_i, the pad turns out to be $H\big(i, \mathbf{b}_i \oplus (\mathbf{s} \odot (\mathbf{c}_{r_i} \oplus \mathbf{c}_j))\big)$ where $\mathbf{c}_{r_i}$ and $\mathbf{c}_j$ are distinct codewords. Since the distance of WH code $\mathcal{C}_{\mathtt{WH}}^\kappa$ is $\kappa/2$, $\mathbf{c}_{r_i}$ and $\mathbf{c}_j$ are different at $\kappa/2$ positions implying that $\kappa/2$ bits of $\mathbf{s}$ contribute to the input of the random oracle H. Since the vector $\mathbf{s}$ is unknown to the receiver (recall that the sender picks $\mathbf{s}$), it is hard for an PPT receiver to retrieve the other pads making the protocol secure for a sender. The protocol of KK13 that realizes $\binom{n}{1}$-OT_ℓ^m given ideal access to $\binom{2}{1}$-OT_m^κ appears in Fig. 3.1. It is easy to verify that the protocol is correct (i.e., $\mathbf{z}_i = \mathbf{x}_{i,r_i}$) when both parties follow the protocol.

3.2 An Attack

At the heart of the attack lies a clever way of manipulating the $\mathbf{E}$ matrix (cf. Section 3.1) which should contain WH codewords in its rows in an honest execution. Recall that the security of the sender lies in the fact that the distance of WH code $\mathcal{C}_{\mathtt{WH}}^\kappa$ is $\kappa/2$. The pads for the messages that are not chosen as the output by the receiver, are the random oracle outputs of an input consisting of $\kappa/2$ bits of $\mathbf{s}$. Since the receiver R does not know $\mathbf{s}$, it cannot guess the pads too in polynomial time. So one way of breaking the privacy of the other inputs of the sender is to

find out the bits of the vector $\mathbf{s}$. Our strategy allows the receiver to recover the ith bit of $\mathbf{s}$ at the cost of two calls to the random oracle under the assumption that R has apriori knowledge of its chosen input $\mathbf{x}_{i,r_i}$ for the ith extended OT. This is achieved by tweaking the rows of $\mathbf{E}$

Figure 3.1: The KK13 OT Extension Protocol

Protocol for $\binom{n}{1}$-$\mathbf{OT}_\ell^m$ from $\binom{2}{1}$-$\mathbf{OT}_m^\kappa$

- **Input of S:** m tuples $\left\{(\mathbf{x}_{i,1}, \cdots, \mathbf{x}_{i,n})\right\}_{i\in[m]}$ of ℓ bit strings.
- **Input of R:** m selection integers $(r_1, \cdots, r_m)$ such that each $r_i \in [n]$.
- **Common Inputs:** A security parameter κ such that $\kappa \geq n$, and Walsh-Hadamard code $\mathcal{C}_{\mathtt{WH}}^\kappa = (\mathbf{c}_1, \cdots, \mathbf{c}_\kappa)$.
- **Oracles and Cryptographic Primitives:** A random oracle $H : [m] \times \{0,1\}^\kappa \to \{0,1\}^\ell$. An ideal $\binom{2}{1}$-OT_m^κ primitive.

1. **Seed OT Phase:**

   (a) S chooses $\mathbf{s} \leftarrow \{0,1\}^\kappa$ at random.

   (b) R forms two $m \times \kappa$ matrices $\mathbf{B}$ and $\mathbf{D}$ in the following way:

   - Choose $\mathbf{b}_i, \mathbf{d}_i \leftarrow \{0,1\}^\kappa$ at random such that $\mathbf{b}_i \oplus \mathbf{d}_i = \mathbf{c}_{r_i}$. Let $\mathbf{E} := \mathbf{B} \oplus \mathbf{D}$. Clearly $\mathbf{e}_i = \mathbf{c}_{r_i}$.

   (c) S and R interact with $\binom{2}{1}$-OT_m^κ in the following way.

   - S acts as *receiver* with input $\mathbf{s}$.
   - R acts as *sender* with input $\left\{(\mathbf{b}^j, \mathbf{d}^j)\right\}_{j\in[\kappa]}$.
   - S receives output $\{\mathbf{a}^j\}_{j\in[\kappa]}$ and forms $m \times \kappa$ matrix $\mathbf{A}$ with the jth column of $\mathbf{A}$ as $\mathbf{a}^j$. Clearly

     i. $\mathbf{a}^j = \big(\mathbf{b}^j \oplus (s_j \odot \mathbf{e}^i)\big)$ and
     ii. $\mathbf{a}_i = \big(\mathbf{b}_i \oplus (\mathbf{s} \odot \mathbf{e}_i)\big) = \big(\mathbf{b}_i \oplus (\mathbf{s} \odot \mathbf{c}_{r_i})\big)$.

2. **OT Extension Phase:**

   (a) For every $i \in [m]$, S computes $\mathbf{y}_{i,j} = \mathbf{x}_{i,j} \oplus H\big(i, \mathbf{a}_i \oplus (\mathbf{s} \odot \mathbf{c}_j)\big)$ and sends $\{\mathbf{y}_{i,j}\}_{j\in[n]}$.

   (b) For every $i \in [m]$, R recovers $\mathbf{z}_i = \mathbf{y}_{i,r_i} \oplus H(i, \mathbf{b}_i)$.

matrix which are codewords from $\mathcal{C}_{\mathtt{WH}}^\kappa$ in an honest execution. Specifically, the ith row of $\mathbf{E}$, $\mathbf{e}_i$ is $\mathbf{c}_{r_i}$ in an honest execution. It is now tweaked to a κ-bit string that is same as $\mathbf{c}_{r_i}$ in all the positions barring the ith position. Specifically, recall that a WH codeword $\mathbf{c}_i$ from $\mathcal{C}_{\mathtt{WH}}^\kappa$ is a κ-length bit vector $(c_{i,1}, \ldots, c_{i,\kappa})$. We denote complement of a bit b by $\overline{b}$. Then the ith row $\mathbf{e}_i$

of $\mathbf{E}$ is set as $(c_{i,1}, \ldots, \overline{c_{i,i}}, \ldots, c_{i,\kappa})$. The matrix is tweaked as above for every ith row as long as $i \leq \kappa$. The rest of the rows in $\mathbf{E}$ starting from κ to m do not need to be tweaked. The matrix $\mathbf{E}$ after tweaking is given below. We denote the tweaked matrix as $\overline{\mathbf{E}}$ and the tweaked rows as $\overline{\mathbf{c}_{r_i}}$ for $i \leq \kappa$.

$$\overline{\mathbf{E}} = \begin{bmatrix} \overline{c_{r_1,1}} & c_{r_1,2} & \ldots & \ldots & \ldots & c_{r_1,\kappa} \\ c_{r_2,1} & \overline{c_{r_2,2}} & \ldots & \ldots & \ldots & c_{r_2,\kappa} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{r_i,1} & c_{r_i,2} & \ldots & \overline{c_{r_i,i}} & \ldots & c_{r_i,\kappa} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{r_\kappa,1} & c_{r_\kappa,2} & \ldots & c_{r_\kappa,j} & \ldots & \overline{c_{r_\kappa,\kappa}} \\ c_{r_\kappa,1} & c_{r_\kappa,2} & \ldots & c_{r_{(\kappa+1)},j} & \ldots & c_{r_{(\kappa+1)},\kappa} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ c_{m,1} & c_{m,2} & \ldots & \ldots & \ldots & c_{m,\kappa} \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{c}_{r_1}} \\ \overline{\mathbf{c}_{r_2}} \\ \vdots \\ \overline{\mathbf{c}_{r_i}} \\ \vdots \\ \overline{\mathbf{c}_{r_\kappa}} \\ \mathbf{c}_{r_{\kappa+1}} \\ \vdots \\ \mathbf{c}_{r_m} \end{bmatrix}$$

When R uses $\overline{\mathbf{E}}$ instead of $\mathbf{E}$, the ith row of $\mathbf{A}$ for $i \leq \kappa$ will be $\overline{\mathbf{a}_i} = (\mathbf{b}_i \oplus (\mathbf{s} \odot \overline{\mathbf{c}_{r_i}}))$. The pad used to mask the r_ith message $\mathbf{x}_{i,r_i}$ in ith extended OT is:

$$\begin{aligned} H\big(i, \overline{\mathbf{a}_i} \oplus (\mathbf{s} \odot \mathbf{c}_{r_i})\big) &= H\Big(i, \mathbf{b}_i \oplus \big(\mathbf{s} \odot (\overline{\mathbf{c}_{r_i}} \oplus \mathbf{c}_{r_i})\big)\Big) \\ &= H\Big(i, \mathbf{b}_i \oplus (\mathbf{s} \odot 0^{i-1}10^{\kappa-i}\Big) \\ &= H\left(i, \mathbf{b}_i \oplus 0^{i-1}s_i0^{\kappa-i}\right) \end{aligned}$$

Now note that the malicious receiver has cleverly made the pad used for $\mathbf{x}_{i,r_i}$ a function of sole unknown bit s_i. With the knowledge of its chosen input $\mathbf{x}_{i,r_i}$ and the padded message $\mathbf{y}_{i,r_i}$ that the receiver receives in the OT extension protocol, the malicious receiver R recovers the value of the pad by finding $\mathbf{y}_{i,r_i} \oplus \mathbf{x}_{i,r_i}$. It further knows that $\mathbf{y}_{i,r_i} \oplus \mathbf{x}_{i,r_i}$ is same as $H\left(i, \mathbf{b}_i \oplus 0^{i-1}s_i0^{\kappa-i}\right)$. Now two calls to the random oracle H with inputs $\left\{(i, \mathbf{b}_i \oplus 0^{i-1}s_i0^{\kappa-i})\right\}_{s_i \in \{0,1\}}$ is sufficient to find the value of s_i. In the similar way, it can find entire input vector of the sender, $\mathbf{s}$ with 2κ number (polynomial in κ) of calls to the random oracle breaking the privacy of the sender completely. The attack works in the version of KK13 that reduces $\binom{n}{1}$-OT_ℓ^m to $\binom{2}{1}$-$\mathsf{OT}_\kappa^\kappa$ without any modification.

3.3 Efficiency of [KK13]

Since efficiency is the prime focus of this paper and we build an OT extension protocol in KK13 style secure against malicious adversaries, we recall the communication complexity of KK13 from [KK13]. For complexity analysis we consider the version of KK13 that reduces $\binom{n}{1}$-OT_ℓ^m to $\binom{2}{1}$-$\mathsf{OT}_\kappa^\kappa$ (presented in Appendix D of [KK13]) and requires less communication than the one that reduces $\binom{n}{1}$-OT_ℓ^m to $\binom{2}{1}$-OT_κ^m. The communication complexity of KK13 OT extension producing $\binom{n}{1}$-OT_ℓ^m is $\mathcal{O}(m(\kappa + n\ell))$ bits.

The best known semi-honest OT extension protocol before KK13 is IKNP protocol [IKNP03] which has a communication complexity of $\mathcal{O}(m(\kappa+\ell))$ bits for producing $\binom{2}{1}$-OT_ℓ^m from $\binom{2}{1}$-$\mathsf{OT}_\kappa^\kappa$. To get $\binom{n}{1}$-OT_ℓ^m as the output from IKNP protocol, the efficient transformation of [NP05] is used. The transformation generates $\binom{n}{1}$-OT_ℓ^1 from $\binom{2}{1}$-$\mathsf{OT}_\kappa^{\log n}$ with an additional (outside the execution of $\binom{2}{1}$-$\mathsf{OT}_\kappa^{\log n}$) communication cost of $\mathcal{O}(\ell n)$ bits. This transformation can be repeated m times to reduce $\binom{n}{1}$-OT_ℓ^m to $\binom{2}{1}$-$\mathsf{OT}_\kappa^{m \log n}$ with an additional communication cost of $\mathcal{O}(\ell m n)$ bits. So to get $\binom{n}{1}$-OT_ℓ^m as the output from IKNP protocol, first $\binom{2}{1}$-$\mathsf{OT}_\kappa^{m \log n}$ is produced via [IKNP03] and then the reduction from $\binom{n}{1}$-OT_ℓ^m to $\binom{2}{1}$-$\mathsf{OT}_\kappa^{m \log n}$ is used that requires an additional communication cost of $\mathcal{O}(\ell m n)$ bits. So the total communication turns out to be $\mathcal{O}(m \log n \cdot (\kappa + \kappa) + \ell m n) = \mathcal{O}(m(\kappa \log n + n\ell))$ bits. Now recall that $n \leq \kappa$, a restriction that comes from the KK13 OT extension (due to the fact that $\mathcal{C}_{\mathtt{WH}}^\kappa$ contains κ codewords). Given this bound, as long as $\ell = \Omega(\log n)$, KK13 OT extension gives better communication complexity than IKNP protocol.

Chapter 4

Actively Secure OT Extension for Short Secrets

We make the KK13 OT extension protocol secure against a malicious receiver by adding a consistency check that relies on linearity of WH code and adds a communication of $\mathcal{O}(\mu \log \kappa)$ bits irrespective of the number of extended OTs. We first discuss the properties of WH code relevant to us for the correctness of the consistency check. We then discuss the check and our actively secure protocol. As we will see the check involves an additional trick apart from the linearity of WH codes to achieve the claimed communication complexity. We also describe the required ideal functionalities. Finally, we show an interesting application of our protocol, namely OT-based Private Set Intersection (PSI).

4.1 Randomized Linearity Testing

We focus on WH code that maps messages of length $\log \kappa$ to codewords of length κ. A WH codeword for a $\log \kappa$-bit input $\mathbf{x}$ can be viewed as a truth table of a linear function $\mathcal{L}_{\mathbf{x}} : \{0,1\}^{\log \kappa} \rightarrow \{0,1\}$ parametrised with $\mathbf{x}$ where $\mathcal{L}_{\mathbf{x}}(\mathbf{a}) = \mathbf{x} \otimes \mathbf{a}$. The WH codeword for $\mathbf{x}$ can be defined as $\mathtt{WH}(\mathbf{x}) := (\mathcal{L}_{\mathbf{x}}(\mathbf{a}))_{\mathbf{a} \in \{0,1\}^{\log \kappa}}$. It is easy to note that $\mathcal{L}_{\mathbf{x}}(\mathbf{a}) = \mathcal{L}_{\mathbf{a}}(\mathbf{x})$ for any $\mathbf{a} \in \{0,1\}^{\log \kappa}$. So we can rewrite the WH codeword for $\mathbf{x}$ as $\mathtt{WH}(\mathbf{x}) := (\mathcal{L}_{\mathbf{a}}(\mathbf{x}))_{\mathbf{a} \in \{0,1\}^{\log \kappa}}$. It is also easy to note that $\mathcal{L}_{\mathbf{a}}()$ is a linear function since $\mathcal{L}_{\mathbf{a}}(\mathbf{x} \oplus \mathbf{y}) = \mathcal{L}_{\mathbf{a}}(\mathbf{x}) \oplus \mathcal{L}_{\mathbf{a}}(\mathbf{y})$ for any $\mathbf{x}$ and $\mathbf{y}$ in $\{0,1\}^{\log \kappa}$. This implies that given codewords, say $\mathbf{c}_{\mathbf{x}}$ and $\mathbf{c}_{\mathbf{y}}$ corresponding to $\mathbf{x}$ and $\mathbf{y}$ respectively, the codeword for $\mathbf{x} \oplus \mathbf{y}$ can be obtained as $\mathbf{c}_{\mathbf{x}} \oplus \mathbf{c}_{\mathbf{y}}$. In general, any linear combination of a set of WH codewords will lead to a WH codeword. On the other hand XOR of a codeword and a non-codeword will be a non-codeword. We note that the above statements are true for pruned code $\mathtt{PRN}_{\mathcal{I}}(\mathcal{C}^{\kappa}_{\mathtt{WH}})$ for any $\mathcal{I}$ of size less than $\kappa/2$. The distance of $\mathtt{PRN}_{\mathcal{I}}(\mathcal{C}^{\kappa}_{\mathtt{WH}})$ is

$\kappa/2 - |\mathcal{J}|$ which is at least 1.

In our OT extension protocol, we need to verify whether a set strings are individually valid WH codewords or not. In particular the number of strings to be verified is proportional to the number of extended OTs output by the OT extension protocol. In practice, it will be in the order of millions. Individual string testing may inflate the computation and the communication cost many-fold. We take the following route to bypass the efficiency loss. Given ν strings for validity verification, we compress them to one string via linear combination taken using a uniform random vector of length ν and then check the compressed string *only* for validity. We show that the compression process ensures that the output string will be a non-codeword with probability at least $\frac{1}{2}$ if the input set contains some non-codeword(s). Below we present the randomized linearity test for ν strings in Fig 4.1 and its probability analysis in Theorem 4.1.1.

Figure 4.1: A Randomized Linearity Test for Many Strings

Randomized Linearity Test for ν Strings

– **Input:** ν κ-bit strings $\mathbf{y}_1, \ldots, \mathbf{y}_\nu$.

– **Output:** `Accept` or `Reject` indicating whether the strings $\mathbf{y}_1, \ldots, \mathbf{y}_\nu$ passes the test or not.

1. **Selection of Random Combiners:** Choose ν bits $b_1, \ldots, b_\nu$ uniformly at random.
2. **The test:** Compute $\mathbf{y} = \bigoplus_{i=1}^{\nu} b_i \odot \mathbf{y}_i$. Output `Accept` if $\mathbf{y}$ is a valid WH codeword, output `Reject` otherwise.

Theorem 4.1.1 *Assume that some of the ν κ-bit strings $\mathbf{y}_1, \ldots, \mathbf{y}_\nu$ are not WH codewords. The randomized linearity test presented in Fig 4.1 outputs* `Reject` *with probability at least $\frac{1}{2}$.*

Proof. Without loss of generality, let $i_1, \ldots, i_\eta$ denote the indices of the input strings that are non-codewords. That is, $\{i_1, \ldots, i_\eta\} \subseteq \{1, \ldots, \nu\}$ and $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$ are exactly the non-codeword strings among the set of ν input strings. It is easy to verify that any linear combination of the remaining strings that are codewords will result in a codeword. So we concentrate on the linear combination that can result from the non-codewords $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$. Let the uniform random bits used to find the linear combination of the non-codewords be $b_{i_1}, \ldots, b_{i_\eta}$. There are 2^η possibilities in total for these η bits which can be interpreted as numbers in the set $\{0, \ldots, 2^\eta - 1\}$. We divide these 2^η strings or numbers in two sets, say $\mathcal{A}$ and $\mathcal{B}$. $\mathcal{A}$ and $\mathcal{B}$ consist of all the strings that corresponds to even and odd numbers respectively from $\{0, \ldots, 2^\eta - 1\}$. Clearly $|\mathcal{A}| = |\mathcal{B}| = 2^{\eta-1}$. We now show that at least $2^{\eta-1}$ strings lead to a non-codeword when they are used as linear combiners for the set of non-codewords $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$. We prove our

claim by showing that for every element in set A, there exists at least one unique string that when used for linear combination of the non-codewords will lead to a non-codeword. Consider a string $\mathbf{w}$ from set A. We have two cases to consider:
(i) $\mathbf{w}$ when used as the linear combiner for $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$ yields a non-codeword. In this case $\mathbf{w}$ itself is the string and element in A that when used as the linear combiner for the non-codewords will lead to a non-codeword.
(ii) $\mathbf{w}$ when used as a linear combiner for $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$ yields a codeword. Note that $\mathbf{w}$ is a string that denotes an even number, say p in $\{0, \ldots, 2^\eta - 1\}$. The least significant bit of $\mathbf{w}$ is a zero. The string corresponding to $p+1$ will belong to the set $\mathcal{B}$ and will have the same form as $\mathbf{w}$ except that the least significant bit will be 1. The linear combination of $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_{\eta-1}}$ with respect to $\mathbf{w}$ is a codeword. We exclude $\mathbf{y}_{i_\eta}$ from the list since the least significant bit of $\mathbf{w}$ is zero. Whereas $\mathbf{y}_{i_\eta}$ is a non-codeword and will be included in the linear combination with respect to the string corresponding to $p+1$.

Clearly, the string corresponding to $p+1$ will lead to a non-codeword as the linear combination of a codeword and a non-codeword always gives a non-codeword. We have shown that for every $\mathbf{w}$ that leads to a codeword, there is a unique string in $\mathcal{B}$ that leads to a non-codeword. The mapping is one-to-one.

We can now conclude that at least half the possibilities of $b_{i_1}, \ldots, b_{i_\eta}$ leads to a non-codeword when used as a linear combiner. Since the linear combiners are chosen uniformly at random, the probability that the linear combination that will result from the non-codewords $\mathbf{y}_{i_1}, \ldots, \mathbf{y}_{i_\eta}$ is a non-codeword is at least $\frac{1}{2}$. Recall that any linear combination of the remaining strings that are codewords will result in a codeword. So the compressed string $\mathbf{y}$ resulted from the linear combination of all the ν strings will be a non-codeword with at least $\frac{1}{2}$ probability too. ■

It is easy to note that the above theorem holds true for $\mathtt{PRN}_{\mathcal{I}}(\mathcal{C}^{\kappa}_{\mathtt{WH}})$ for any $\mathcal{I}$ of size less than $\kappa/2$. So we get the following corollary.

Corollary 4.1.2 *Let $\mathcal{I} \subset [\kappa]$ be a set of size less than $\kappa/2$. Assume that some of the ν $\kappa - |\mathcal{I}|$-bit vectors $\mathbf{y}_1, \ldots, \mathbf{y}_\nu$ are not pruned WH codewords. Then $\mathbf{y} \notin \mathtt{PRN}_{\mathcal{I}}(\mathcal{C}^{\kappa}_{WH})$ with probability at least $\frac{1}{2}$ where $\mathbf{y} = \bigoplus_{i=1}^{\nu} b_i \odot \mathbf{y}_i$ and the bits $b_1, \ldots, b_\nu$ are uniform random.*

4.2 Functionalities

We describe the ideal functionalities that we need. Below we present an OT functionality parameterized using three parameters ℓ that denotes the string length of the sender's inputs, n that refers to 1-out-of-n OTs and m that denotes the number of instances of the OTs.

Next we present a functionality to generate uniformly random common coins.

Figure 4.2: The Ideal Functionality for $\binom{n}{1}$-OT_ℓ^m

Functionality $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$

$\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$ interacts with S, R and the adversary $\mathcal{S}$ and is parameterized by three parameters ℓ that denotes the string length of the sender's inputs, n that refers to 1-out-of-n OTs and m that denotes the number of instances of the OTs.

- Upon receiving m tuples $\left\{(\mathbf{x}_{i,1}, \cdots, \mathbf{x}_{i,n})\right\}_{i\in[m]}$ of ℓ bit strings from S and m selection integers $(r_1, \cdots, r_m)$ such that each $r_i \in [n]$ from R, the functionality sends $\left\{\mathbf{x}_{i,r_i}\right\}_{i\in[m]}$ to R. Otherwise it aborts.

Figure 4.3: The Ideal Functionality for generating random common coins

Functionality $\mathcal{F}_{\mathbf{COIN}}$

$\mathcal{F}_{\mathrm{COIN}}$ interacts with S, R and the adversary $\mathcal{S}$.

- Upon receiving $(\mathtt{coin}, \ell)$ from both S and R, the functionality generates ℓ random bits, say $\mathbf{w}$ and sends $\mathbf{w}$ to both S and R. Otherwise it aborts.

4.3 The Protocol

We now describe the protocol that realizes $\binom{n}{1}$-OT_ℓ^m given ideal access to $\binom{2}{1}$-$\mathsf{OT}_\kappa^\kappa$. The protocol is similar to the protocol of KK13 (cf. Fig. 3.1), except that our protocol includes a consistency check for preventing R from behaving maliciously and using non-codewords in matrix $\mathbf{E}$. The check makes use of the Randomized Linearity Testing described in Section 4.1. It is trivial to see that Randomized Linearity Test alone doesn't suffice, since a malicious R can provide some vector for the check independent from what he had used in the seed OTs. Thus we need a check to ensure that the vector provided by R for the check is consistent with the vectors used in the seed OTs. We make use of the fact that if both S and R are honest, then we have $\mathbf{a}_i = \mathbf{b}_i \oplus (\mathbf{e}_i \odot \mathbf{s})$. A closer analysis of this expression gives a simple verification mechanism for a corrupt R. Namely, R sends to S a random linear combination of the rows of $\mathbf{B}$ and $\mathbf{E}$, say $\mathbf{b}$ and $\mathbf{e}$ respectively, for a commonly agreed random linear combiner generated using a coin tossing protocol. S then applies the same random linear combiner on the rows of $\mathbf{A}$ to obtain $\mathbf{a}$ and checks if $\mathbf{b}$ and $\mathbf{e}$ are consistent with $\mathbf{s}$ and $\mathbf{a}$. Namely, whether $\mathbf{a} = \mathbf{b} \oplus (\mathbf{e} \odot \mathbf{s})$ holds or not. While the above check is simple, it requires communication of κ-bit vectors, namely $\mathbf{b}$ and $\mathbf{e}$. The communication is brought down to $\mathcal{O}(\log \kappa)$ using a couple of tricks. First, a second level of compression function is applied on $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{e} \odot \mathbf{s}$ via xor on the bits of the individual vectors. This results in three bits a, b and p respectively from $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{e} \odot \mathbf{s}$. Then the check is simply to verify if $a = b \oplus p$. Notice that $\mathbf{e} \odot \mathbf{s}$ can be perceived as the linear combination of $\mathbf{e}$ for random combiner $\mathbf{s}$. Since $\mathbf{s}$ is privy to S, R cannot compute the linear combination of $\mathbf{e} \odot \mathbf{s}$,

namely p. So R sends across the index of the codeword that matches with $\mathbf{e}$ and on receiving it S computes p after computing $\mathbf{e} \odot \mathbf{s}$. The index requires just $\log \kappa$ bits as $\mathcal{C}^{\kappa}_{\mathtt{WH}}$ consists of κ codewords. Thus our final consistency check needs communication of $\mathcal{O}(\log \kappa)$ bits and a sequence of cheap xor operations. Lastly, the above check is repeated μ times, where μ denotes the statistical security parameter. We show that either a corrupt R tweaks few positions of the codewords allowing error-correction or it is caught. Either event takes place with overwhelming probability. Looking ahead to the proof, the former event allows the simulator to extract the inputs of corrupted R and thereby making the real and the ideal world indistinguishable with high probability. Whereas, the protocol is aborted in both the real and ideal worlds when the latter event happens. Our construction appears in Fig. 4.4.

4.3.1 Security

The correctness of our protocol follows from the correctness of the KK13 protocol and the correctness of the consistency check. While the former is explained in Section 3.1, the latter is explained below. The linearly combined vectors $\mathbf{e}^{(l)}$ for $l \in [\mu]$ will be valid codewords follows directly from the linearity of WH code as mentioned in Section 4.1. When R is honest we have $\mathbf{a}_i = \mathbf{b}_i \oplus (\mathbf{e}_i \odot \mathbf{s})$ and $\mathbf{c}_{\alpha^{(l)}} = \mathbf{e}^{(l)}$ for $l \in [\mu]$. Thus, for every $l \in [\mu]$,

$$\begin{aligned}
\mathbf{a}^{(l)} &= \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{a}_i = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot [\mathbf{b}_i \oplus (\mathbf{e}_i \odot \mathbf{s})] \\
&= \left[\bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{b}_i\right] \oplus \left[\left(\bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{e}_i\right) \odot \mathbf{s}\right] \\
&= \mathbf{b}^{(l)} \oplus (\mathbf{e}^{(l)} \odot \mathbf{s}) \\
a^{(l)} &= \bigoplus_{i=1}^{\kappa} a_i^{(l)} = \bigoplus_{i=1}^{\kappa} (b_i^{(l)} \oplus (s_i \odot e_i^{(l)})) = b^{(l)} \oplus p^{(l)}
\end{aligned} \tag{4.1}$$

Now it is easy to verify that the protocol is correct (i.e., $\mathbf{z}_i = \mathbf{x}_{i,r_i}$) when both the parties follow the protocol.

We now move on to the security argument for our protocol. The original OT extension of [KK13] provides security against a malicious S. Since our consistency check involves message communication from R to S, it does not offer any new scope for a malicious sender to cheat. However, the check may reveal some information about R's input. Recall that R's input is encoded in the rows of matrix $\mathbf{E}$ and during the check, a random linear combination of the rows of $\mathbf{E}$ (where the combiner is known to S) is presented to S for verification. The check is repeated for μ times. To prevent information leakage on R's input, $\mathbf{E}$ is padded with μ extra

Figure 4.4: Actively Secure OT Extension Protocol

Protocol for $\binom{n}{1}$-$\mathbf{OT}_\ell^m$ from $\binom{2}{1}$-$\mathbf{OT}_\kappa^\kappa$

- **Input of S:** m tuples $\big\{(\mathbf{x}_{i,1}, \cdots, \mathbf{x}_{i,n})\big\}_{i\in[m]}$ of ℓ bit strings.
- **Input of R:** m selection integers $(r_1, \cdots, r_m)$ such that each $r_i \in [n]$.
- **Common Inputs:** A security parameter κ such that $\kappa \geq n$, and Walsh-Hadamard code $\mathcal{C}^\kappa_{\mathtt{WH}} = (\mathbf{c}_1, \cdots, \mathbf{c}_\kappa)$.
- **Oracles, Cryptographic Primitives and Functionalities:** A random oracle $H : [m] \times \{0,1\}^\kappa \to \{0,1\}^\ell$ and a pseudorandom generator $G : \{0,1\}^\kappa \to \{0,1\}^{m+\mu}$. An ideal OT functionality $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$ and an ideal coin tossing functionality $\mathcal{F}_{\mathrm{COIN}}$.

1. **Seed OT Phase:**
   (a) S chooses $\mathbf{s} \leftarrow \{0,1\}^\kappa$ at random.
   (b) R chooses κ pairs of seeds $(\mathbf{k}_j^0, \mathbf{k}_j^1)$ each of length κ.
   (c) S and R interact with $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$ in the following way.
   - S acts as *receiver* with input $\mathbf{s}$.
   - R acts as *sender* with input $\big\{(\mathbf{k}_i^0, \mathbf{k}_i^1)\big\}_{i\in[\kappa]}$.
   - S receives output $\{\mathbf{k}_i^{s_i}\}_{i\in[\kappa]}$.
2. **OT Extension Phase I:**
   (a) R forms three $(m+\mu)\times\kappa$ matrices $\mathbf{B}$, $\mathbf{E}$ and $\mathbf{D}$ in the following way and sends $\mathbf{D}$ to S:
   - Set $\mathbf{b}^j = G(\mathbf{k}_j^0)$.
   - Set $\mathbf{e}_i = \mathbf{c}_{r_i}$ for $i \in [m]$. For $i \in [m+1, m+\mu]$, set $\mathbf{e}_i$ to a randomly picked codeword from $\mathcal{C}^\kappa_{\mathtt{WH}}$.
   - Set $\mathbf{d}^j = \mathbf{b}^j \oplus G(\mathbf{k}_j^1) \oplus \mathbf{e}^j$.

   (b) On receiving $\mathbf{D}$, S forms $(m+\mu)\times\kappa$ matrix $\mathbf{A}$ with the jth column of $\mathbf{A}$ set as $\mathbf{a}^j = \big(s_j \odot \mathbf{d}^j\big) \oplus G(\mathbf{k}_j^{s_j})$. Clearly, (i) $\mathbf{a}^j = \big(\mathbf{b}^j \oplus (s_j \odot \mathbf{e}^j)\big)$ and (ii) $\mathbf{a}_i = \big(\mathbf{b}_i \oplus (\mathbf{s}\odot\mathbf{e}_i)\big) = \big(\mathbf{b}_i \oplus (\mathbf{s}\odot\mathbf{c}_{r_i})\big)$.
3. **Checking Phase:**
   (a) S and R invoke $\mathcal{F}_{\mathrm{COIN}}$ with $(\mathtt{coin}, \mu(m+\mu))$ and receives an μ $(m+\mu)$-length random bit vectors say $\mathbf{w}^{(1)}, \ldots, \mathbf{w}^{(\mu)}$. On receiving the vectors, the parties do the following for $l \in [\mu]$:
   - R computes $\mathbf{b}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{b}_i$, $\mathbf{e}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{e}_i$ and $b^{(l)} = \bigoplus_{i=1}^{\kappa} b_i^{(l)}$. It sends $b^{(l)}$ and $\alpha^{(l)}$ where $\mathbf{e}^{(l)} = \mathbf{c}_{\alpha^{(l)}}$ to S.
   - S computes $\mathbf{a}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{a}_i$, $a^{(l)} = \bigoplus_{i=1}^{\kappa} a_i^{(l)}$, $\mathbf{p}^{(l)} = \mathbf{s} \odot \mathbf{c}_{\alpha^{(l)}}$ and $p^{(l)} = \bigoplus_{i=1}^{\kappa} p_i^{(l)}$. It aborts the protocol if $a^{(l)} \neq b^{(l)} \oplus p^{(l)}$.
4. **OT Extension Phase II:**
   (a) For every $i \in [m]$, S computes $\mathbf{y}_{i,j} = \mathbf{x}_{i,j} \oplus H\big(i, \mathbf{a}_i \oplus (\mathbf{s} \odot \mathbf{c}_j)\big)$ and sends $\{\mathbf{y}_{i,j}\}_{j\in[n]}$.
   (b) For every $i \in [m]$, R recovers $\mathbf{z}_i = \mathbf{y}_{i,r_i} \oplus H(i, \mathbf{b}_i)$.

rows consisting of random codewords. This ensures that the linear combination presented in an instance of the check will look random and will bear no information about the m rows of $\mathbf{E}$ that encode R's input, unless the bits of the random combiner corresponding to the padded μ rows are zero. However, the probability of that happening is only $\frac{1}{2^\mu}$.

A corrupt R can cheat by not picking the rows of $\mathbf{E}$ as codewords. Our consistency check ensures an overwhelming probability for catching such a misconduct when 'large' number of positions in the codewords are tweaked. If few positions are tweaked, then we show that the tweaked codewords are error-correctable with high probability allowing the simulator in the proof to extract input of the corrupt R. We now prove security formally.

Theorem 4.3.1 *The protocol in Fig. 4.4 securely realizes $\mathcal{F}_{OT}^{(n,m,\ell)}$ in the $(\mathcal{F}_{OT}^{(2,\kappa,\kappa)}, \mathcal{F}_{\text{COIN}})$-hybrid model.*

Proof. Our proof is presented in Universal Composability (UC) framework recalled briefly in Section 2.4.1. To prove the security of our protocol, we describe two simulators. The simulator $\mathcal{S}_\mathsf{S}$ simulates the view of a corrupt sender and appears in Fig. 4.5. On the other hand, the simulator $\mathcal{S}_\mathsf{R}$ simulates the view of a corrupt receiver and is presented in Fig. 4.6.

We now prove that $\text{IDEAL}_{\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)},\mathcal{S}_\mathsf{S},\mathcal{Z}} \overset{c}{\approx} \text{REAL}_{\binom{n}{1}\text{-}\mathsf{OT}_\ell^m,\mathcal{A},\mathcal{Z}}$ when $\mathcal{A}$ corrupts S. In $(\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}, \mathcal{F}_{\text{COIN}})$-hybrid model, we note that the difference between the simulated and the real view lies in $\mathbf{D}$ matrix. In the simulated world, the matrix $\mathbf{D}$ is a random matrix, whereas in the real world it is a pseudo-random matrix. The indistinguishability can be proved via a reduction to PRG security.

Next, we prove that $\text{IDEAL}_{\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)},\mathcal{S}_\mathsf{R},\mathcal{Z}} \overset{c}{\approx} \text{REAL}_{\binom{n}{1}\text{-}\mathsf{OT}_\ell^m,\mathcal{A},\mathcal{Z}}$ when $\mathcal{A}$ corrupts R via a series of hybrids. The output of each hybrid is always just the output of the environment $\mathcal{Z}$. Starting with $\mathbf{HYB}_0 = \text{REAL}_{\binom{n}{1}\text{-}\mathsf{OT}_\ell^m,\mathcal{A},\mathcal{Z}}$, we gradually make changes to define $\mathbf{HYB}_1$ and $\mathbf{HYB}_2$ as follows:

$\mathbf{HYB}_1$: Same as $\mathbf{HYB}_0$, except that in the **Checking Phase**, the protocol is aborted when the simulator $\mathcal{S}_\mathsf{R}$ fails to extract the input of R.

$\mathbf{HYB}_2$: Same as $\mathbf{HYB}_1$, except that default value 0^ℓ is substituted for the inputs $\left\{\mathbf{x}_{i,j}\right\}_{i\in[m]\wedge j\neq r_i}$

Clearly, $\mathbf{HYB}_2 = \text{IDEAL}_{\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)},\mathcal{S}_\mathsf{R},\mathcal{Z}}$. Our proof will conclude, as we show that every two consecutive hybrids are computationally indistinguishable.

$\mathbf{HYB}_0 \overset{c}{\approx} \mathbf{HYB}_1$: The difference between $\mathbf{HYB}_0$ and $\mathbf{HYB}_1$ lies in the condition on aborting the protocol. In $\mathbf{HYB}_0$ the protocol is aborted when $a^{(l)} \neq b^{(l)} \oplus p^{(l)}$ for some $l \in [\mu]$ (cf. Fig. 4.4). Whereas, in $\mathbf{HYB}_1$ the protocol is aborted when either the condition for abortion in $\mathbf{HYB}_0$ is true or the extraction fails. The latter implies that either $|\mathcal{T}| \geq \kappa/2$ or there exist an

Figure 4.5: Simulator $\mathcal{S}_{\mathsf{S}}$ for Malicious Sender

Simulator $\mathcal{S}_{\mathsf{S}}$ for S

The simulator plays the role of the honest R and simulates each step of the protocol $\binom{n}{1}$-OT_{ℓ}^{m} as follows. The communication of the $\mathcal{Z}$ with the adversary $\mathcal{A}$ who corrupts S is handled as follows: Every input value received by the simulator from $\mathcal{Z}$ is written on $\mathcal{A}$'s input tape. Likewise, every output value written by $\mathcal{A}$ on its output tape is copied to the simulator's output tape (to be read by the environment $\mathcal{Z}$).

1. **Seed OT Phase:** On behalf of $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$, $\mathcal{S}_{\mathsf{S}}$ receives $\mathbf{s}$, the input of S to the functionality $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$. Next it picks κ PRG seeds $\mathbf{k}_i$ each of length κ and sends $\{\mathbf{k}_i\}_{i\in[\kappa]}$ to S on behalf of $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$.

2. **OT Extension Phase I:** $\mathcal{S}_{\mathsf{S}}$ picks a $(m+\mu)\times\kappa$ matrix $\mathbf{D}$ uniformly at random and sends to S. It then computes matrix $\mathbf{A}$ using the PRG seeds sent to S, $\mathbf{s}$ and $\mathbf{D}$. Namely, it sets $\mathbf{a}^j = \big(s_j \odot \mathbf{d}^j\big) \oplus G(\mathbf{k}_j)$.

3. **Checking Phase:** On receiving $(\mathtt{coin}, \mu(m+\mu))$ from S on behalf of $\mathcal{F}_{\mathrm{COIN}}$, $\mathcal{S}_{\mathsf{S}}$ sends μ $(m+\mu)$-length random bit vectors say $\mathbf{w}^{(1)},\ldots,\mathbf{w}^{(\mu)}$. For $l\in[\mu]$, it then computes $\mathbf{a}^{(l)}$ and $a^{(l)}$ using $\mathbf{w}^{(l)}$ and $\mathbf{A}$ just as an honest S does. It chooses a random WH codeword, say $\mathbf{e}^{(l)}$, sets $\mathbf{b}^{(l)} = \mathbf{a}^{(l)} \oplus (\mathbf{s}\odot\mathbf{e}^{(l)})$ and computes $b^{(l)}$ using $\mathbf{b}^{(l)}$. Finally, it sends $\alpha^{(l)}$, the index of $\mathbf{e}^{(l)}$ and $b^{(l)}$ to S.

4. **OT Extension Phase II:** On receiving $\{(\mathbf{y}_{i,1},\cdots,\mathbf{y}_{i,n})\}_{i\in[m]}$ from S, $\mathcal{S}_{\mathsf{S}}$ computes $\mathbf{x}_{i,j} = \mathbf{y}_{i,j} \oplus H\big(i, \mathbf{a}_i \oplus (\mathbf{s}\odot\mathbf{c}_j)\big)$ for $1\le i\le m$ and sends $\{(\mathbf{x}_{i,1},\cdots,\mathbf{x}_{i,n})\}_{i\in[m]}$ to functionality $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$ on behalf of S.

index i such that $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}_i) \notin \mathtt{PRN}_{\mathcal{T}}(\mathtt{C}_{\mathtt{WH}}^{\kappa})$. Let PC denote the event of passing the consistency check for a corrupt R who commits a non-codeword matrix $\mathbf{E}$ in the seed OT phase. Let FE denote the event of failed extraction of R's input. Lastly, let D denote the event that $\mathcal{Z}$ distinguishes between $\mathbf{HYB}_0$ and $\mathbf{HYB}_1$. Then, we have $\mathsf{Pr}[\mathsf{D} \mid \neg\mathsf{PC}] = 0$ (since the execution aborts in both hybrids) and $\mathsf{Pr}[\mathsf{D} \mid \mathsf{PC}] = \mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}]$. So we have,

$$\begin{aligned}\mathsf{Pr}[\mathsf{D}] &= \mathsf{Pr}[\mathsf{D} \mid \mathsf{PC}]\cdot\mathsf{Pr}[\mathsf{PC}] + \mathsf{Pr}[\mathsf{D} \mid \neg\mathsf{PC}]\cdot\mathsf{Pr}[\neg\mathsf{PC}] \\ &= \mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}]\cdot\mathsf{Pr}[\mathsf{PC}]\end{aligned} \tag{4.2}$$

We now show that $\mathsf{Pr}[\mathsf{D}]$ is negligible in κ and μ because either the probability of passing the check is negligible or the probability of failure in extraction when check has passed is negligible. In other words, we show that $\mathsf{Pr}[\mathsf{PC}] \le \mathtt{negl}(\kappa,\mu)$ when $|\mathcal{T}| \ge \kappa/2$ and $\mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}] \le \mathtt{negl}(\kappa,\mu)$

otherwise. We capture the above in the following two lemmas.

Figure 4.6: Simulator $\mathcal{S}_{\mathsf{R}}$ for Malicious Receiver

Simulator $\mathcal{S}_{\mathsf{R}}$ for R.

The simulator plays the role of the honest S and simulates each step of the protocol $\binom{n}{1}$-OT_{ℓ}^{m} as follows. The communication of the $\mathcal{Z}$ with the adversary $\mathcal{A}$ who corrupts R is handled as follows: Every input value received by the simulator from $\mathcal{Z}$ is written on $\mathcal{A}$'s input tape. Likewise, every output value written by $\mathcal{A}$ on its output tape is copied to the simulator's output tape (to be read by the environment $\mathcal{Z}$).

1. **Seed OT Phase:** On behalf of $\mathcal{F}_{\mathsf{OT}}^{(2,\kappa,\kappa)}$, $\mathcal{S}_{\mathsf{R}}$ receives the input of R to the functionality, namely $\left\{(\mathbf{k}_i^0, \mathbf{k}_i^1)\right\}_{i\in[\kappa]}$.

2. **OT Extension Phase I:** On receiving matrix $\mathbf{D}$ from R, $\mathcal{S}_{\mathsf{R}}$ computes $\mathbf{E}$ using the knowledge of $\left\{(\mathbf{k}_i^0, \mathbf{k}_i^1)\right\}_{i\in[\kappa]}$. That is, it computes $\mathbf{E}$ as $\mathbf{e}^i = G(\mathbf{k}_i^0) \oplus G(\mathbf{k}_i^1) \oplus \mathbf{d}^i$, where $i \in [\kappa]$.

3. **Checking Phase:** On receiving $(\mathtt{coin}, \mu(m+\mu))$ from R on behalf of $\mathcal{F}_{\mathrm{COIN}}$, $\mathcal{S}_{\mathsf{R}}$ sends μ $(m+\mu)$-length random bit vectors say $\mathbf{w}^{(1)}, \ldots, \mathbf{w}^{(\mu)}$ to R. Then $l \in [\mu]$, it receives $b^{(l)}$ and $\alpha^{(l)}$ from R and performs the consistency check honestly like an honest S. If the check fails, then it sends $\mathtt{Abort}$ to $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$ on behalf of R and halts. If none of the check fails, $\mathcal{S}_{\mathsf{R}}$ computes $\mathbf{e}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{e}_i$ using the rows of $\mathbf{E}$ and finds $\mathcal{T}_i = \mathtt{HDI}(\mathbf{e}^{(l)}, \mathbf{c}_{\alpha^{(l)}})$ for $l \in [\mu]$. It then computes $\mathcal{T} = \bigcup_{l=1}^{\mu} \mathcal{T}_l$. If $|\mathcal{T}| \geq \kappa/2$ or there exists an index i such that $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}_i) \notin \mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$[a], then it sends $\mathtt{Abort}$ to $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$. Otherwise, $\mathcal{S}_{\mathsf{R}}$ extracts the ith input of R as r_i where $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}_i) = \mathtt{PRN}_{\mathcal{T}}(\mathbf{c}_{r_i})$ for $i \in [m]$.

4. **OT Extension Phase II:** $\mathcal{S}_{\mathsf{R}}$ sends the input of R, namely $(r_1, \cdots, r_m)$ (such that each $r_i \in [n]$) to functionality $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$ on behalf of R. From $\mathcal{F}_{\mathsf{OT}}^{(n,m,\ell)}$, it receives $\left\{\mathbf{x}_{i,r_i}\right\}_{i\in[m]}$. It then runs the protocol with R using $\left\{\mathbf{x}_{i,r_i}\right\}_{i\in[m]}$ and 0^{ℓ} for the unknown inputs $\left\{\mathbf{x}_{i,j}\right\}_{i\in[m]\wedge j\neq r_i}$.

[a] Note that $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ consists of κ vectors with distance $\kappa/2 - |\mathcal{T}|$ which is at least one when $|\mathcal{T}| < \kappa/2$. This follows from the fact that the distance of $\mathcal{C}_{\mathtt{WH}}^{\kappa}$ is $\kappa/2$.

Lemma 4.3.2 $\mathsf{Pr}[\mathsf{D}] \leq \max(\frac{1}{2^{|\mathcal{T}|}}, \frac{1}{2^{\mu}})$, *when* $|\mathcal{T}| \geq \kappa/2$.

Proof. When $|\mathcal{T}| \geq \kappa/2$, we note that $\mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}] = 1$ as the extraction always fails. Plugging the equality in Equation 4.2, we get $\mathsf{Pr}[\mathsf{D}] = \mathsf{Pr}[\mathsf{PC}]$. Next we conclude the proof by showing that $\mathsf{Pr}[\mathsf{PC}] = \max(\frac{1}{2^{|\mathcal{T}|}}, \frac{1}{2^{\mu}})$ which is negligible in κ and μ.

Consider lth iteration of the check in **Checking Phase**. Recall that $a^{(l)}$ at S's end is computed as follows. First $\mathbf{a}^{(l)}$ is calculated as $\mathbf{a}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{a}_i$ where $\mathbf{a}_i = \big(\mathbf{b}_i \oplus (\mathbf{s} \odot \mathbf{e}_i)\big)$.

Denoting $\mathbf{b}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{b}_i$ and $\mathbf{e}^{(l)} = \bigoplus_{i=1}^{m+\mu} w_i^{(l)} \odot \mathbf{e}_i$, we have $\mathbf{a}^{(l)} = \mathbf{b}^{(l)} \oplus (\mathbf{s} \odot \mathbf{e}^{(l)})$. Lastly, denoting $b^{(l)} = \bigoplus_{i=1}^{\kappa} b_i^{(l)}$ and $p = \bigoplus_{i=1}^{\kappa} s_i \odot e_i^{(l)}$, we have $a^{(l)} = \bigoplus_{i=1}^{\kappa} a_i^{(l)} = b^{(l)} \oplus p^{(l)}$. Let a corrupt R sends the index $\alpha^{(l)}$. Let $\bar{b}^{(l)}$ denote the bit sent along with $\alpha^{(l)}$ and let $\bar{p}^{(l)} = \bigoplus_{i=1}^{\kappa} s_i \odot c_i^{(l)}$ where $\mathbf{c}_{\alpha^{(l)}} = [c_1^{(l)}, \ldots, c_\kappa^{(l)}]$. Now the check passes when $b^{(l)} \oplus p^{(l)} = \bar{b}^{(l)} \oplus \bar{p}^{(l)}$. The equation implies that

$$\begin{aligned} b^{(l)} \oplus \bar{b}^{(l)} &= p^{(l)} \oplus \bar{p}^{(l)} = \left(\bigoplus_{i=1}^{\kappa} s_i \odot e_i^{(l)} \right) \bigoplus \left(\bigoplus_{i=1}^{\kappa} s_i \odot c_i^{(l)} \right) \\ &= \bigoplus_{i=1}^{\kappa} s_i \odot (e_i^{(l)} \oplus c_i^{(l)}) \end{aligned}$$

Now note that the bits of $\mathbf{s}$ corresponding to the indices *not* in $\mathcal{T}$ do not have any impact on the value of $\bigoplus_{i=1}^{\kappa} s_i \odot (e_i^{(l)} \oplus c_i^{(l)})$. So $2^{\kappa - |\mathcal{T}|}$ possibilities of the vector $\mathbf{s}$ will lead to passing the check. Since $\mathbf{s}$ is chosen uniformly at random and is a κ-length bit vector, the probability that the chosen vector will hit one of the $2^{\kappa-|\mathcal{T}|}$ possibilities is $\frac{2^{\kappa-|\mathcal{T}|}}{2^\kappa}$. The probability of passing the check is thus $\frac{1}{2^{|\mathcal{T}|}}$. Another way of passing the check is to hit the value of $\bar{b}^{(l)}$ in all the μ instances of the check so that the equalities $b^{(l)} \oplus p^{(l)} = \bar{b}^{(l)} \oplus \bar{p}^{(l)}$ for $l \in [\mu]$ hold good. The probability of passing the check in this way thus turns out to be $\frac{1}{2^\mu}$. This concludes the proof. ■

Lemma 4.3.3 $\mathsf{Pr}[\mathsf{D}] \leq \frac{1}{2^\mu}$, *when* $|\mathcal{T}| < \kappa/2$.

Proof. From Equation 4.2, we get the inequality $\mathsf{Pr}[\mathsf{D}] \leq \mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}]$. We now show that $\mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}] \leq \frac{1}{2^\mu}$. We note that when $|\mathcal{T}| < \kappa/2$, the reason for failure in extraction happens because some of the pruned rows of $\mathbf{E}$ do not belong to the the pruned code $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$. That is, there exists an index i such that $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}_i) \notin \mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$. Now the fact that the distance of $\mathcal{C}_{\mathtt{WH}}^{\kappa}$ is $\kappa/2$ and the number of indices that are pruned are strictly less that $\kappa/2$ implies that $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ consists of κ vectors with distance $\kappa/2 - |\mathcal{T}|$ which is at least one. Now Corollary 4.1.2 implies that if some of the pruned rows of $\mathbf{E}$ do not belong to $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$, then $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}^{(l)})$ belongs to $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ with probability at most $1/2$. Since $\mathbf{e}^{(l)}$s are computed using independent and uniformly picked random linear combiners, at least one of $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}^{(1)}), \ldots, \mathtt{PRN}_{\mathcal{T}}(\mathbf{e}^{(\mu)})$ do not belong to $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ with probability at least $1 - \frac{1}{2^\mu}$. Recall that $\mathbf{e}^{(l)}$ is computed using $\mathbf{w}^{(l)}$. But since $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}^{(l)}) = \mathtt{PRN}_{\mathcal{T}}(\mathbf{c}_{\alpha^{(l)}})$ and $\mathtt{PRN}_{\mathcal{T}}(\mathbf{c}_{\alpha^{(l)}}) \in \mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ for all $l \in [\mu]$, it implies that $\mathtt{PRN}_{\mathcal{T}}(\mathbf{e}^i)$ for all $i \in [m+\mu]$ belong to $\mathtt{PRN}_{\mathcal{T}}(\mathcal{C}_{\mathtt{WH}}^{\kappa})$ with probability at least $1 - \frac{1}{2^\mu}$. So we have

$\mathsf{Pr}[\neg\mathsf{FE} \mid \mathsf{PC}] \geq 1 - \frac{1}{2^\mu}$ which implies $\mathsf{Pr}[\mathsf{FE} \mid \mathsf{PC}] \leq \frac{1}{2^\mu}$ ■

$\mathbf{HYB}_1 \overset{c}{\approx} \mathbf{HYB}_2$: The difference between $\mathbf{HYB}_1$ and $\mathbf{HYB}_2$ lies in the values for the inputs $\{\mathbf{x}_{i,j}\}_{i\in[m]\wedge j\neq r_i}$. In $\mathbf{HYB}_1$ these values are the real values of an honest S whereas in $\mathbf{HYB}_2$ these are the default value 0^ℓ. The security in this case will follow from the random oracle assumption of H. We proceed in two steps. First, assume that the distinguisher of $\mathbf{HYB}_1$ and $\mathbf{HYB}_2$ does not make any query to H. We show that the pads used to mask the unknown inputs of S will be uniformly random and independent of each other due to random oracle assumption. Recall that the pads for masking $\{\mathbf{x}_{i,j}\}_{i\in[m]\wedge j\neq r_i}$ are $\{H\big(i, \mathbf{b}_i \oplus (\mathbf{s} \odot (\mathbf{c}_{r_i} \oplus \mathbf{c}_j))\big)\}_{i\in[m]\wedge j\neq r_i}$. Since $\mathcal{C}^\kappa_{\mathtt{WH}}$ is a WH code, the Hamming weight of each vector in the set $\{(\mathbf{c}_{r_i} \oplus \mathbf{c}_j)\}_{i\in[m]\wedge j\neq r_i}$ is at least $\kappa/2$. Since $\mathbf{s}$ is picked at random from $\{0,1\}^\kappa$, each of the values in $\{\mathbf{s} \odot (\mathbf{c}_{r_i} \oplus \mathbf{c}_j)\}_{i\in[m]\wedge j\neq r_i}$ is uniformly distributed over a domain of size at least $2^{\kappa/2}$. Now random oracle assumption lets us conclude that the pads $\{H\big(i, \mathbf{b}_i \oplus (\mathbf{s} \odot (\mathbf{c}_{r_i} \oplus \mathbf{c}_j))\big)\}_{i\in[m]\wedge j\neq r_i}$ are random and independent of each other and thus provide information-theoretic blinding guarantee to the values $\{\mathbf{x}_{i,j}\}_{i\in[m]\wedge j\neq r_i}$.

Next, following the standard of proofs in the random oracle model we allow the distinguisher to make polynomial (in κ) number of adaptive queries to H. Clearly, if a distinguisher makes a query to H on any of the values $\{\big(i, \mathbf{b}_i \oplus (\mathbf{s} \odot (\mathbf{c}_{r_i} \oplus \mathbf{c}_j))\big)\}_{i\in[m]\wedge j\neq r_i}$ that are used to mask the unknown inputs of S, then it can distinguish between the hybrids. Such queries are denoted as *offending queries*. As long as no offending query is made, each of these $m(n-1)$ offending queries is (individually) distributed uniformly at random over a domain of size (at least) $2^{\kappa/2}$ and so the distinguisher's probability of hitting upon an offending query remains the same as in the case he does not make any query at all to H. So if the distinguisher makes q queries, then it's probability of distinguishing only increases by a polynomial factor over $2^{-\kappa/2}$. ■

Our security proof relies on random oracle assumption of H. However, as mentioned in [KK13], the random oracle assumption can be replaced with a generalized notion of correlation-robustness [IKNP03] referred as C-correlation-robustness [KK13] in a straightforward way. For completeness, we recall the definition of C-correlation-robust hash functions below.

Definition 4.3.4 ([KK13]) *Let $C = \{\mathbf{c}_1, \ldots, \mathbf{c}_n\}$ be a set of κ-bit strings such $n = \mathsf{Poly}(\kappa)$ and for any j, k with $j \neq k$, the Hamming distance between $\mathbf{c}_j$ and $\mathbf{c}_k$ is $\Omega(\kappa)$. Then a hash function $H : \{0,1\}^* \to \{0,1\}^{\ell(\kappa)}$ is C-correlation-robust if for any polynomial $m(\kappa)$ and any non-uniform PPT distinguisher $\mathcal{A}$ provided with input C has $\mathtt{negl}(\kappa)$ probability of distinguishing the following distributions:*

- $\Big\{\{\big(j, k, H(i, \mathbf{b}_i \oplus ((\mathbf{c}_j \oplus \mathbf{c}_k) \odot \mathbf{s}))\big)\}_{i\in[m],j,k\in[n],j\neq k}\Big\}$ *where each string in $\{\mathbf{b}_i\}_{i\in[m]}$ is a κ-bit and $\mathbf{s}$ is a κ-bit string chosen uniformly at random and independent of $\{\mathbf{b}_i\}_{i\in[m]}$.*

– $U_{m(n-1)\ell}$; $U_{m(n-1)\ell}$ *denotes uniform distribution over* $\{0,1\}^{m(n-1)\ell}$.

4.3.2 Efficiency

The actively secure protocol incurs a communication of $\mathcal{O}(\kappa^2)$ bits in **Seed OT Phase**. In **OT Extension Phase I**, R sends $\kappa(m+\mu)$ bits to S. In **Checking Phase**, S and R invokes $\mathcal{F}_{\text{COIN}}$. We follow the implementation of [KOS15] for $\mathcal{F}_{\text{COIN}}$ that generates $\mu(m+\mu)$ bits at one go and uses a pseudorandom function (PRF) and a PRG. Let $F_k : \{0,1\}^{2\kappa} \to \{0,1\}^{\kappa}$ be a keyed PRF with $k \in \{0,1\}^{\kappa}$ be a uniform random string and $G : \{0,1\}^{\kappa} \to \{0,1\}^{\mu(m+\mu)}$ be a PRG. Then $\mathcal{F}_{\text{COIN}}$ can be realized as follows:

1. R generates and sends random $s_{\mathsf{R}} \leftarrow \{0,1\}^{\kappa}$ to S.
2. S generates and sends random $s_{\mathsf{S}} \leftarrow \{0,1\}^{\kappa}$ to R.
3. Both parties compute $s = F_k(s_{\mathsf{S}}, s_{\mathsf{R}})$ and output $\mathbf{w}_1||\mathbf{w}_2 \ldots ||\mathbf{w}_\mu = G(s)$ where each $\mathbf{w}_i \in \{0,1\}^{(m+\mu)}$.

With the above implementation of $\mathcal{F}_{\text{COIN}}$, **Checking Phase** incurs a communication of $\mathcal{O}(\mu \log \kappa)$. In **OT Extension Phase II**, S communicates $mn\ell$ bits to R. So the total communication our protocol is $\mathcal{O}(\kappa^2 + \kappa(m+\mu) + \mu \log \kappa + mn\ell) = \mathcal{O}(m(\kappa + n\ell))$ (assuming m is asymptotically bigger than κ and μ) bits which is same as that of KK13 OT extension.

Computation-wise, **Checking Phase** constitutes the additional work that our protocol does over KK13 protocol. The additional work involves cheap xor and bit-wise multiplications.

4.4 Empirical Results

We compare our work with the existing protocols of KK13 [KK13], IKNP [IKNP03], ALSZ15 [ALSZ15] and NNOB [NNOB12] in terms of communication and runtime in LAN and WAN settings. The implementation of KOS [KOS15] is not available in the platform that we consider for benchmarking. As per the claim made in KOS, the runtime of their OT extension bears an overhead of 5% with respect to IKNP protocol both in LAN and WAN. The communication complexity of KOS is at least the complexity of IKNP. These results allow to get a clear idea on how KOS fares compared to our protocol.

In any practical scenario, the computation is not the prime bottleneck, as computing power has improved a lot due to improvements in hardwares. The communication overhead is the main issue, and so most of the aforementioned protocols are aimed at improving the communication complexity. Our empirical results show that our proposed protocol performs way better than even the passively secure IKNP in terms of communication complexity when $\binom{n}{1}$-OTs with short input are expected outcomes. Since ALSZ15, NNOB and KOS are built upon IKNP,

they lag behind our protocol in performance too. Though our prime focus is to improve the communication complexity, our protocol outperforms IKNP and the existing actively secure protocols in runtime both in LAN and WAN setting. We now detail the software, hardware and implementation specifications used in our empirical analysis before presenting our experimental findings.

Software Details. We build upon the OT extension code provided by the Encrypto group on github [OTC]. It contains the OT extension implementations of KK13, NNOB and ALSZ15 in C++, using the Miracl library for elliptic curve arithmetic. We build upon the KK13 code for our actively secure protocol. AES-128 has been used for the PRG instantiation and the random oracle has been implemented by the SHA-256 hash function.

Hardware Details. We have tested the code in a standard LAN network and a simulated WAN setting. Our machine has 8 GB RAM and an Intel Core i5-4690 CPU with 3.5 GHz processor speed. For WAN simulation, we used the tc tool of Linux, where we introduced a round trip delay of 100 milliseconds into the network, with a limited bandwidth of 20 Mbps.

Implementation Details. We discuss our choice of m, n and ℓ denoting the number of extended output OTs, the number of inputs of S in each extended OT and the bit length of S's input respectively. In other words, we refer to the parameters m, n and ℓ of $\binom{n}{1}$-OT_ℓ^m. Recall that as long as the input length of S, namely ℓ satisfies the relation $\ell = \Omega(\log n)$, theoretically KK13 OT extension (and our proposed OT extension) gives better communication complexity for producing $\binom{n}{1}$-OT_ℓ^m than IKNP protocol and its variants (cf. Section 3.3). For benchmarking, we take two approaches. First, we fix $n = 16$ and $\ell = 4(= \log 16)$ and experiment on the following values of m: 1.25×10^5, 2.5×10^5, 5×10^5 and 1.25×10^6. Next, we fix m to a value and vary n from 8 to 256 in the powers of 2. The value of ℓ for each choice of n is set as $\log n$. Our protocol and KK13 directly generate OTs of type $\binom{n}{1}$-OT_ℓ whereas IKNP, ALSZ15 and NNOB generate OTs of type $\binom{2}{1}$-OT_1. To compare with IKNP, ALSZ15 and NNOB, we convert the output OTs of these protocols, namely $\binom{2}{1}$-OT_1 to $\binom{n}{1}$-OT_ℓ using the efficient transformation of [NP05] (cf. Section 3.3).

To obtain a computational security guarantee of 2^{-128}, while KK13 and our protocol need 256 seed OTs, IKNP, NNOB and ALSZ15 need 128, 342 and respectively 170 seed OTs. Among these, except IKNP and KK13, the rest are maliciously secure. To obtain a statistical security guarantee of 2^{-40} against a malicious receiver, ALSZ15 and NNOB need 380 checks whereas we need 96 checks.

We follow the approach of ALSZ15 implementation and perform the OT extension in batches of 2^{16} in sequential order. For each batch, the sender and the receiver perform the seed OTs, participate in a coin tossing protocol, perform the checks and finally obtain the output. We use one thread in the sender as well as in the receiver side in order to calculate the upper bound on the computation cost. However our code is compatible with multiple threads where each thread can carry out a batch of OTs. Lastly, our seed OT implementation relies on the protocol of [PVW08].

4.4.1 Performance Comparison

Since we build upon KK13 protocol, we first display in Table 4.1-4.2 the overhead (in %) of our protocol compared to KK13. Notably, the communication overhead lies in the range **0.011**%-**0.028**%. Table 4.1 shows that for large enough number of extended OTs, the runtime overhead of our protocol over KK13 ranges between 4-6% for both LAN and WAN. Table 4.2 demonstrates that the runtime overheads drop below 2% when in addition the number of inputs of the sender in the extended OTs is large enough.

Next, our empirical results are shown in Table 4.3-4.4 and Fig. 4.7-4.8. First, we discuss our results in Table 4.3 and Fig. 4.7 where we vary m. Next, we focus on the results displayed in Table 4.4 and Fig. 4.8 where we vary n. In both the case studies, our protocol turns out to be the best choice among the actively secure OT extensions and second best overall closely trailing KK13 which is the overall winner. Communication complexity wise, our protocol is as good as KK13 and is way better than the rest. The empirical results are in concurrence with the theoretical $\log n$ improvement of KK13 and our protocol over IKNP (and its variants).

Table 4.1: Runtime and Communication Overhead (in %) of Our protocol over KK13 for producing $\binom{16}{1}$-OT_4^m.

	Runtime		Communication
m	LAN	WAN	
1.25×10^5	6.48	9.27	0.012
2.5×10^5	6.33	8.76	0.012
5×10^5	5.88	7.09	0.012
1.25×10^6	3.78	5.72	0.028

Performance Comparison for varied m values. The results in Table 4.3 reflects that KK13 is the best performer in terms of communication as well as runtime in LAN and WAN. Our actively secure protocol is the second best closely trailing KK13. Our protocol has communication overhead of 0.012-0.028% over KK13, while IKNP, ALSZ15 and NNOB have overheads of

Table 4.2: Runtime and Communication Overhead (in %) of Our protocol over KK13 for producing $\binom{n}{1}$-$\mathsf{OT}^{10^6}_{\log n}$.

n	Runtime		Communication
	LAN	WAN	
8	6.16	11.77	0.011
16	4.13	6.6	0.012
32	4.5	2.29	0.013
64	3.65	1.81	0.014
128	1.24	1.18	0.015
256	0.58	0.8	0.015

79.7-84%, 249% and 352% respectively. Noticeably, we observe that the cost for generating 5×10^5 $\binom{16}{1}$-OT using our protocol is less than the cost of generating 1.25×10^5 $\binom{16}{1}$-OT using NNOB. Similarly the cost of generating 1.25×10^6 $\binom{16}{1}$-OT using our protocol is 71.6% of the cost of generating 5×10^5 $\binom{16}{1}$-OT using ALSZ15.

In LAN setting, the overheads over KK13 vary in the range of 3.78-6.48%, 11-17.60%, 14.4-22.7% and 14.5-20.8% respectively for our protocol, IKNP, ALSZ15 and NNOB. The similar figures in WAN setting are 5.72-9.26%, 16-22.6%, 35.3-39% and 24.1-29.1% respectively for our protocol, IKNP, ALSZ15 and NNOB. A pictorial representation is shown in Fig. 4.7.

Table 4.3: Performance Comparison of various OT extensions for producing $\binom{16}{1}$-OT^m_4.

m	Runtime in LAN (in sec)					Runtime in WAN (in sec)					Communication (in MB)				
	KK13	**This paper**	**IKNP**	**ALSZ15**	**NNOB**	**KK13**	**This paper**	**IKNP**	**ALSZ15**	**NNOB**	**KK13**	**This paper**	**IKNP**	**ALSZ15**	**NNOB**
1.25×10^5	02.16	02.30	02.54	02.65	02.61	13.38	14.62	16.40	18.10	16.90	04.77	04.77	08.66	16.67	21.60
2.5×10^5	04.23	04.50	04.88	05.26	05.05	24.32	26.45	29.26	33.80	31.40	09.54	09.54	17.15	33.33	43.21
5×10^5	08.50	09.00	09.78	10.04	10.10	47.39	50.75	56.9	65.00	60.40	19.08	19.08	34.79	66.62	86.39
1.25×10^6	21.68	22.50	24.07	24.81	24.84	115.34	121.94	133.81	158.60	143.20	47.69	47.70	87.74	166.54	215.95

Figure 4.7: Performance Comparison of various OT extensions for producing $\binom{16}{1}$-OT^m_4.

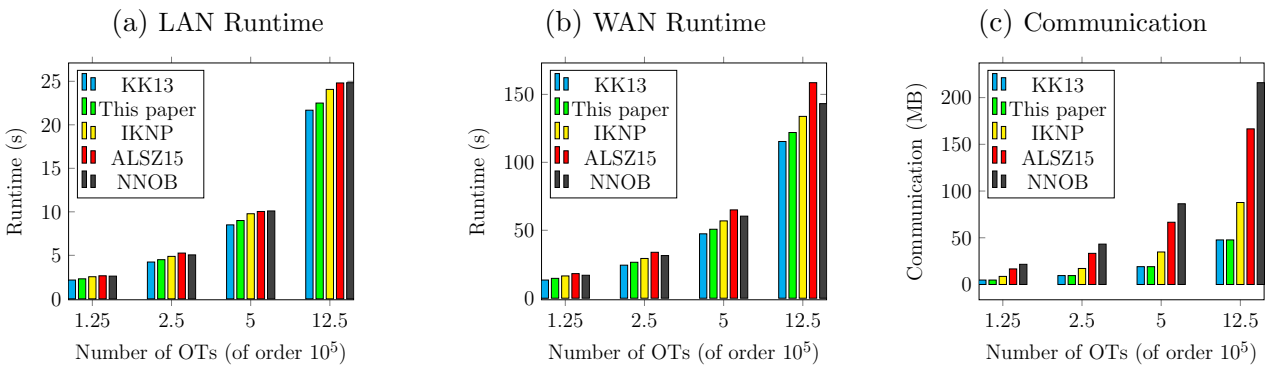
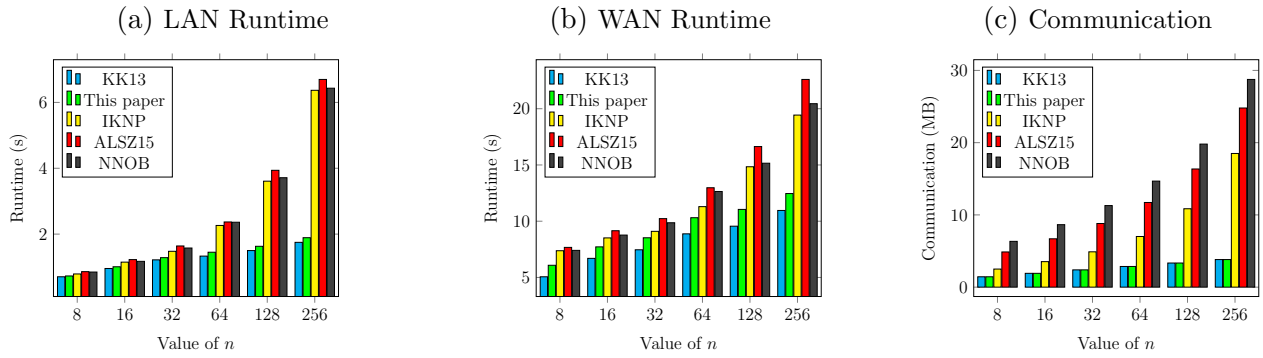


Table 4.4: Performance Comparison of various OT extensions for producing $\binom{n}{1}$-$\mathsf{OT}^{5\times10^4}_{\log n}$.

n	Runtime in LAN (in sec)					Runtime in WAN (in sec)					Communication (in MB)				
	KK13	**This paper**	**IKNP**	**ALSZ15**	**NNOB**	**KK13**	**This paper**	**IKNP**	**ALSZ15**	**NNOB**	**KK13**	**This paper**	**IKNP**	**ALSZ15**	**NNOB**
8	0.70	0.73	0.79	0.86	0.85	5.06	6.08	7.38	7.67	7.41	1.43	1.43	2.5	4.87	6.34
16	0.96	1.01	1.15	1.23	1.17	6.70	7.72	8.52	9.15	8.77	1.91	1.91	3.53	6.69	8.65
32	1.22	1.28	1.48	1.64	1.58	7.46	8.53	9.10	10.2	9.86	2.39	2.39	4.89	8.81	11.29
64	1.33	1.45	2.26	2.37	2.36	8.88	10.31	11.29	12.97	12.64	2.86	2.86	7.01	11.7	14.68
128	1.50	1.63	3.61	3.94	3.71	9.56	11.05	14.84	16.63	15.16	3.34	3.34	10.85	16.36	19.8
256	1.75	1.89	6.37	6.70	6.43	10.96	12.46	19.43	22.6	20.45	3.82	3.82	18.5	24.8	28.75

Figure 4.8: Performance Comparison of various OT extensions for producing $\binom{n}{1}$-$\mathsf{OT}^{5\times10^4}_{\log n}$.



Performance Comparison for varied n values. Here we set $m = 5\times10^4$ and vary n from 8 to 256 in the powers of 2. Similar to the previous case study, KK13 turns out the best performer here as well (cf. Table 4.4). Our protocol is the second best closely trailing KK13. Our protocol has communication overhead of 0.011-0.028% over KK13, while IKNP, ALSZ15 and NNOB have overheads of 74.5-384.6%, 239.5-549.4% and 341.8-652.9% respectively. In LAN setting, the overheads over KK13 vary in the range of 3.5-8.8%, 12.2-263.8%, 22.2-282.7% and 20.5-267.5% respectively for our protocol, IKNP, ALSZ15 and NNOB. The similar figures in WAN setting are 13.7-20.2%, 27.1-77.4%, 36.7-106.3% and 30.9-86.6% respectively for our protocol, IKNP, ALSZ15 and NNOB. A pictorial representation is shown in Fig. 4.8.

In the following section, we describe one of the interesting applications of our OT Extension protocol.

4.5 Application to Private Set Intersection

Our protocol can be used in the OT based Private Set Intersection (PSI) protocol of [Lam16] to obtain the most efficient PSI protocol that is maliciously secure against a corrupt receiver and semi-honestly secure against a corrupt sender. In PSI, a sender S and a receiver R hold sets $X = \{x_1, x_2, \ldots x_{n_1}\}$ and $Y = \{y_1, y_2, \ldots y_{n_2}\}$ respectively. The goal of the protocol is to let the receiver know the intersection $X \cap Y$ and nothing more. Put simply, a PSI protocol

realizes the functionality $\mathcal{F}_{\mathsf{PSI}}(X, Y) = (\perp, X \cap Y)$. The set sizes are assumed to be public.

We set our focus on the PSI protocols that are OT-based so that we can employ our OT extension protocol in them to improve efficiency. [PSZ14] introduced an OT-based PSI protocol relying on black-box usage of random $\binom{n}{1}$-OT. Subsequently, [PSSZ15] improved the communication overhead of [PSZ14]. Both the protocols are semi-honestly secure. At the core of the protocols lies an important building block called set inclusion that allows R to check whether its input, say y, is contained in X, owned by S, while preserving the input privacy of both the parties. In the set inclusion protocol, the receiver breaks its σ-bit element, say y into t blocks of η-bits. Similarly S breaks each of its σ-bit element x_i into t blocks of η-bits. Next, a random $\binom{2^\eta}{1}$-OT is used for kth block of receiver's input for $k = 1, \ldots, t$ where the random OT does the following. Denoting $N = 2^\eta$, a random OT of above type generates N random masks and delivers them to S. R receives from the OT the mask corresponding to its block which acts as its choice string. S then generates a mask for each of its elements in X using the masks received from the t random $\binom{N}{1}$-OTs. Similarly, R combines the masks it receives from the OTs to generate the mask corresponding to its input element y. The verification whether y is included in X is then done by performing checks over the masks. Namely S sends across the masks corresponding to all its elements in X. R verifies if the mask corresponding to y matches with one of them or not. In a naive approach, PSI can be achieved by having the receiver run the set inclusion protocol n_2 times, once for each element in Y. [PSZ14] and subsequently [PSSZ15] improved the complexity of the naive approach by reducing the number of OTs and improving the input length of S in the OTs. Various hashing techniques such as Simple Hashing [PSZ14] and Cuckoo Hashing (with a stash s [KMW09]), h-ary Cuckoo Hashing [FPSS03] and Feistel-like functions [ANS10] were used to achieve the goal. However, as mentioned before, both [PSZ14] and [PSSZ15] works in semi-honest setting. Indeed, Lambæk in his detailed analysis [Lam16] finds three vulnerabilities in [PSZ14, PSSZ15] when malicious adversaries are considered. Details follow.

One of vulnerabilities corresponds to sender corruption. Fixing the problem remains an open question. The remaining two vulnerabilities correspond to the receiver corruption. In more details, the first problem comes from a malicious receiver who can learn whether some elements of its choice outside his set Y of size n_2 belong to S's input X or not. The solution proposed in the thesis to thwart this attack uses Shamir secret sharing (SS) paired with symmetric-key encryption (SKE). Recall that, S sends the masks corresponding to its elements in X after the OT executions to help R identify the elements in the intersection. The idea of the proposed solution of [Lam16] is to lock the masks using a key of SKE, secret share the key and allow R to recover the key only when R uses less than or equal to n_2 elements in the set inclusion

protocols (i.e. in the OT executions). The second vulnerability may result from any malicious behaviour of R in the OT executions of set inclusion protocol. [Lam16] proposes to fix the problem by using maliciously secure (against corrupt receiver) OT protocols. Using off-the-shelves maliciously secure OT extension protocols, [Lam16] therefore obtains a PSI protocol that is maliciously secure against corrupt R but semi-honestly secure against corrupt S. For complete details of the protocol of Lambæk, refer [Lam16].

We propose to use our maliciously secure OT extension protocol in the PSI protocol of [Lam16] to obtain the most efficient PSI protocol that is maliciously secure against corrupt R but semi-honestly secure against corrupt S. As evident from the theoretical and experimental results presented in this work, our maliciously secure OT extension protocol is a better choice compared to the existing maliciously secure extension protocols [ALSZ15, NNOB12, KOS15] when the OTs required are of type $\binom{n}{1}$-OT. As PSI employs $\binom{n}{1}$-OT (instead of $\binom{2}{1}$-OT), our extension protocol fits the bill. Lastly, we find a concrete vulnerability for the malicious corrupt R case in Lambæk's PSI protocol when semi-honest KK13 OT extension is used in it. This confirms Lambæk's concern of privacy breach of his PSI protocol that may result from privacy breach of the underlying OT protocols and further confirms the necessity of maliciously secure OT extension in Lambæk's PSI protocol. The attack by the corrupt R goes as follows: Using the concrete attack discussed in Section 3.1 for KK13 protocol, a corrupt R in the PSI protocol can recover the outputs to S in the OT executions. The outputs to S are used to compute the masks for the elements of X. Therefore by violating the privacy of semi-honest KK13, R can completely recover the masks for all the elements of X bypassing the security of secret sharing technique coupled with SKE. This allows R to learn whether some elements of its choice outside his set Y of size n_2 belong to S's input X or not.

Chapter 5

Summary and Future Work

5.1 Summary of the Thesis

This thesis started by providing an overview of the area of Secure Multi-party Computation and its building block, namely Oblivious Transfer. After presenting some preliminaries and literature survey, we proceeded with our main result. Prior to presenting our protocol, we revisited the efficient protocol on which ours is built upon. We then showed the formal construction of our protocol, followed by a rigorous security proof and the empirical findings. To be more specific:

- Our protocol adds a communication overhead of $\mathcal{O}(\mu \log \kappa)$ bits over KK13 protocol irrespective of the number of extended OTs, where κ and μ refer to computational and statistical security parameter respectively.
- In terms of concrete efficiency, our protocol when used to generate a large enough number of OTs adds only 0.011-0.028% communication overhead and 4-6% runtime overhead both in LAN and WAN over KK13 extension.

Finally, we showed an interesting application of our protocol in OT-based PSI protocols. Specifically, we use our protocol to obtain the most efficient PSI protocol that is maliciously secure against a corrupt receiver and semi-honestly secure against a corrupt sender.

5.2 Directions for Future Work

Owing to the novel nature of the problem studied, there are several natural extensions to this thesis. The following list mentions a few immediate possible directions for future work.

- While we focused on the well-known Walsh-Hadamard codeword for optimizing our consistency check, studying the proposed approach under other linear codewords such as BCH, Golay Codes etc can be a prospective research direction.

- Another related challenging open problem is that of efficiently converting 1-out-of-n OTs to 1-out-of-2 OTs in the malicious setting. Such a solution combined with our protocol would enable us to compare with the state-of-the-art 1-out-of-2 OT Extension protocols.

- In [PSSZ15], Pinkas et al. have used permutation based hashing on [PSZ14] which thereby reduced the runtime by a factor of 10 and the communication by a factor 2.5-3.4 of their PSI protocol. It would be interesting to see whether those optimizations can be applied to the PSI protocol described in the thesis, while preserving the maliciously secure property.

- It would also be interesting to see whether the protocol of Lambæk [Lam16] can be extended to provide active security for the case of malicious sender.

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