E0 337: Topics in Advanced Cryptography 21st Aug, 2023

Lecture 1

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1 Welcome to E0 337

1.1 Course Information

Contact information and office hours:

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1.2 Course Topics

The course will cover the following topics:

- Introduction to Game Theory (Setting and Examples)
- Dominant Strategy (Definition)

2 Introduction to Game Theory

Game theory is the analysis of interactions between strategic agents which may either be rational or intelligent. A "game" can be any scenario where there are multiple different strategies any participating entity (read "player") can follow. Each strategy may result in a different outcome depending on the strategy choice of the other players. Game theory can be applied to anything from the Rock-Paper-Scissors game to geo-political and military scenarios.

A Game can be defined as $G = (N, \Gamma, U)$, where N is the set of players, Γ is the strategy profile and U is the Utility function for a specific set of strategies.

2.1 Players

All participating entities of the game are considered players, denoted by $N = 1, ...n$. All players are generally assumed to have the following properties.

- **Rational:** All parties considered to be rational employ the strategy which is of most benefit to themselves.
- **Intelligent:** All parties considered intelligent make a decision regarding their strategy taking into account all information known by the game theorist and possessing the ability to make the same inferences as a game theorist.

Next, we shall define what a strategy is.

2.2 Strategy

A *Strategy* is a set of choices which is meant to provide a beneficial outcome to a player considering the expected moves of other players as well.

All possible strategies of a player i is denoted by Γ_i

All Strategies employed at a given time by all players is denoted by $S = (S_1, S_2, ..., S_n)$. Strategies of all players except player i is given by $S_{-i} = (S_1, S_2, ..., S_{i-1}, S_{i+1}, ..., S_n)$. Hence, All strategies can also be denoted as $S = (S_i, S_{-i})$

2.3 Utility Functions

All parties have preferences in outcomes which are mapped to numbers for each of comparison. The goal of a rational player in any given scenario is to maximise the Utility they achieve from the game. For any game i with $N = (1, 2, ...n)$ players: $U_i : (\Gamma_1 x \Gamma_2 x ....\Gamma_n) \to R$

For any two Strategy profiles: $U_i((S')) \geq U_i((S))$

Example 1. ***Rock-Paper-Scissors:*** *The Rock-Paper-Scissors game has no draw condition, so only win - or - lose condition; it is modelled as a Zero-Sum game. The possible Utilities between two players A and B for a Rock-Paper-Scissors game is given as follows*

Table 1: Rock-Paper-Scissors Game

A \B	R	P	S
R	(0,0)	(-1,1)	(1,-1)
P	(1,-1)	(0,0)	(-1,1)
S	(-1,1)	(1,-1)	(0,0)

Example 2. ***Welfare-War Game:*** *In this game, two kingdoms A and B are considered to be equivalent in terms of economic and military strength. The rulers of the two countries are given a choice, either strengthen the military of the country (War) or in Welfare schemes. Welfare schemes lead to prosperity and has an equable outcome if both countries invest in welfare. If either counrty invests in war and the other in welfare, it can invade and gain the benefits of welfare as well. If both countries invest in war, then neither get the benefits of welfare schemes. This game is not a win-or-lose game, as both countries investing in the same area will result in a draw. Thus, it is modelled as follows:*

Table 2: Welfare-War Game

A \B	Welfare	War
Welfare	(5,5)	(7,0)
War	(0,7)	(1,1)

Example 3. ***Prisoners' Fiasco:*** *In this game, two criminals A and B are caught by the police and are given two choices, either to confess or stay silent. The sentence of the one who confesses is dropped and the other one serves a very long sentence. If both confess, both serve a sentence. If neither confess, they both serve minimal sentences and are free. Any sentence is considered a loss in Utility and is assigned negative values. Thus, the game is modelled as follows:*

Exercise 4. *Look at the modelling of the* Tragedy of Commons *Game.*

Table 3: Prisoners' Fiasco

A \B	Silent	Confess
Silent	(-1,-1)	(-7,0)
Confess	(0,-7)	(-5,-5)

2.4 Dominant Strategy

Consider Player i, S_i is a dominant strategy if $U_i(S_i, S_{-i}) \geq U_i(S'_i, S_{-i})\ \forall S'_i \in \Gamma_i, \forall S_{-i} \in \Gamma_{-i}$

A Weakly Dominant Strategy is one where for at least some $S^*_{-i} \in \Gamma_{-i}\ U(S_i, S^*_{-i}) > U_i(S'_i,^*_{-i})$

The Dominant strategy is one which provides the best utility irrespective of the opponents' strategy.

For the above Welfare-War game, while it seems optimal that cooperation between the two kingdoms with welfare investments seem like the good choice, all rational players will choose to invest in war. The dominant strategy in this game is investment in war, as it always provides highest utility. When your opponent chooses Welfare, you get higher utility by playing War. When the opponent plays War, you get higher utiltiy by playing War again. Thus, in this game, the dominant strategy is observed to be War all the time.

On the other hand, in a Rock-Paper-Scissors game, there is no single strategy that always provides hgihest utility irrespective of the opponents' strategies. In this game, the lack of a dominant strategy is observed.

In the next lecture, the topic of Equilibrium is covered.

References

[1] A.R. Karlin and Y. Peres. *Game Theory, Alive.* Miscellaneous Book Series. American Mathematical Society, 2017.

[2] Y. Narahari. *Game Theory and Mechanism Design.* G - Reference,Information and Interdisciplinary Subjects Series. IISc Press, 2014.