Linear Bounded Automata

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Overview

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- Definition

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- Results about LBAs

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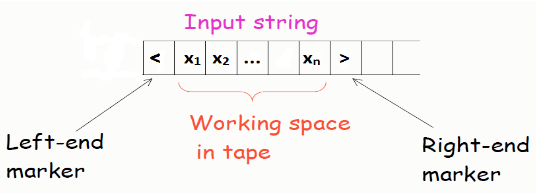
- CSLs and LBAs

Definition

A Turing machine that uses only the tape space occupied by the input is called a linear-bounded automaton (LBA).



- A linear bounded automaton is a nondeterministic Turing machine $M = (Q, \Sigma, \Gamma, \delta, s, t, r)$ such that:
  - There are two special tape symbols $<$ and $>$(the left end marker and right end marker).
  - The TM begins in the configuration $(s, < x >, 0)$.
  - The TM cannot replace $<$ or $>$ with anything else, nor move the tape head left of $<$ or right of $>$.

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- **Examples**: $\{a^n b^n c^n | n \geq 0\}$; counting number of a's
- This limitation makes the LBA a somewhat more accurate model of computers that actually exist than a Turing machine, whose definition assumes unlimited tape.

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- Shortly thereafter, Landweber proved that the languages accepted by a deterministic LBA are always context-sensitive.
- In 1964, Kuroda introduced the more general model of (nondeterministic) linear bounded automata, and showed that the languages accepted by them are precisely the context-sensitive languages.

Number of configurations

- Suppose that a given LBA M has
  - q states,
  - m characters in the tape alphabet ,
  - and the input length is n.
- Then M can be in at most

$$\alpha(n) = \overbrace{m^n}^{Tapecontents} * \overbrace{n}^{Headposition} * \overbrace{q}^{State}$$

configurations.

Results about LBA

Halting Problem

The halting problem is solvable for linear bounded automata.

- $Halt_{LBA} = \{< M, w > |$M is an LBA and M halts on w$\}$ is decidable.
- An LBA that stops on input w must stop in at most $\alpha(|w|)$ steps

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Membership problem

The membership problem is solvable for linear bounded automata.

- $A_{LBA} = \{< M, w > |$M is an LBA and M accepts w$\}$ is decidable.

Results about LBA

Emptiness Problem

The emptiness problem is unsolvable for linear bounded automata.

- For every Turing machine there is a linear bounded automaton which accepts the set of strings which are valid halting computations for the Turing machine.

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   - Open Problem!
2. Is the class of languages accepted by LBA closed under complement?
   - Yes. (*Immerman Szelepcsenyi* Theorem)

LBAs and CSLs

Theorem(Landweber-Kuroda)

A language is accepted by an LBA iff it is context sensitive.

Proof

$\Leftarrow$

If L is a CSL, then L is accepted by some LBA.

- Let $G = (N, \Sigma, S, P)$ be the given grammar such that $L(G) = L$.
- Construct LBA M with tape alphabet $\Sigma \times \{N \cup \Sigma\}$(2- track machine)
- First track holds input string w
- Second track holds a sentential form α of G, initialized to S.

Proof...

- If $w = \epsilon$, M halts without accepting.
- Repeat :
  1. Non-deterministically select a position i in α.
  2. Non-deterministically select a production $\beta \rightarrow \gamma$ of G.
  3. If β appears beginning in position i of α, replace β by γ there.
     - If the sentential form is longer than w, LBA halts without accepting.
  4. Compare the resulting sentential form with w on track 1. If they match, accept. If not go to step 1.

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- *Sketch of proof*:
- Derivation simulates moves of LBA
- Three types of productions
  1. Productions that can generate two copies of a string in Σ^*, along with some symbols that act as markers to keep the two copies separate.
  2. Productions that can simulate a sequence of moves of M. During this portion of a derivation, one of the two copies of the original string is left unchanged; the other, representing the input tape to M, is modified accordingly.
  3. Productions that can erase everything but the unmodified copy of the string, provided that the simulated moves of M applied to the other copy cause M to accept.

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