**Automata Theory and Computability**

*Assignment 3 (Context-Free Grammars)*

(Due on Fri 18 Oct 2013)

1. Consider the grammar $G$ below:

$$S \longrightarrow aS \mid aSbS \mid \epsilon.$$

   (a) Describe the language generated by this grammar.

   (b) Give a formal proof of correctness of your claim.

2. Consider the language $\mathrm{BP}_2$ of "balanced parenthesis" over the alphabet $\{(,),[,]\}$. For example, the string "(()[])" is in the language but not "([)]". Thus $\mathrm{BP}_2$ is similar to BP except that the type of a closing bracket must match the type of the last unmatched opening bracket. Give a CFG for $\mathrm{BP}_2$. There is no need to prove your answer correct.

3. Convert the following context-free grammar to Chomsky Normal Form:

$$\begin{array}{lcl} S & \longrightarrow & ASA \mid aB \\ A & \longrightarrow & B \mid S \\ B & \longrightarrow & b \mid \epsilon \end{array}$$

4. Show that the language

$$L = \{a^n b^{n^2} \mid n \geq 0\}.$$

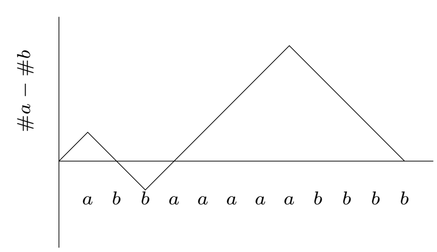
   is not a CFL.

5. Consider the grammar $G$ below:

$$S \longrightarrow SS \mid aSb \mid bSa \mid \epsilon.$$

   Prove that it generates the language $\{x \in \{a,b\}^* \mid \#_a(x) = \#_b(x)\}$.

   *Hint:* Consider the graph of a word $x$ where you plot the value $\#_a(y) - \#_b(y)$ against prefixes $y$ of $x$. Use induction as usual.




6. For the CFG $G$ in Question 5 above answer the following questions.

(a) Give a regular language that is letter-equivalent to $L(G)$. Such a language is guaranteed to exist from Parikh's theorem.

(b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is, first identify the basic pumps for $G$, and the $\leq$-minimal parse trees. Use these to obtain an expression for $\psi(L)$.

7. For $A, B \subseteq \Sigma^*$, define

$$\begin{array}{lcl} A/B & = & \{x \mid \exists y \in B: \; xy \in A\} \\ A \leftarrow B & = & \{x \mid \forall y \in B: \; xy \in A\}. \end{array}$$

Exactly one of the following statements is true. Which one is?

(a) If $L$ is context-free, so is $L/\Sigma^*$.

(b) If $L$ is context-free, so is $L \leftarrow \Sigma^*$.