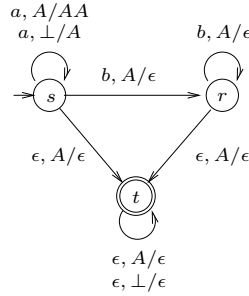


## Automata Theory and Computability

### Assignment 4 (Pushdown Automata)

(Due on Mon 4th Nov 2013)

1. Consider the PDA given below in diagrammatic form:



Thus, the transitions of the PDA are

$$\begin{aligned}
 (s, a, \perp) &\rightarrow (s, A\perp) \\
 (s, a, A) &\rightarrow (s, AA) \\
 (s, b, A) &\rightarrow (r, \epsilon) \\
 (s, \epsilon, A) &\rightarrow (t, \epsilon) \\
 (r, b, A) &\rightarrow (r, \epsilon) \\
 (r, \epsilon, A) &\rightarrow (t, \epsilon) \\
 (t, \epsilon, A) &\rightarrow (t, \epsilon) \\
 (t, \epsilon, \perp) &\rightarrow (t, \epsilon),
 \end{aligned}$$

and the PDA accepts by final state ( $\{t\}$ ).

Describe the language accepted by the PDA.

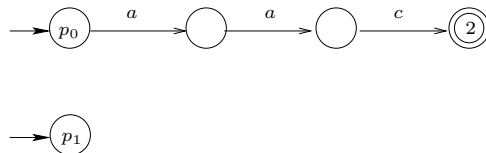
2. Give Pushdown Automata (PDA's) for each of the following languages. If possible, give deterministic PDA's for the languages. Use the diagrammatic representation of Question 1 above. Specify all transitions of the PDA's.

- (a)  $\{wcw^R \mid w \in \{a, b\}^*\}$ .
- (b)  $\{w \in \{a, b\}^* \mid \#_a(w) \geq 2\#_b(w)\}$ .

3. This question is from Wolfgang Thomas's notes on Applied Automata Theory. Consider the pushdown system  $\mathcal{P}$  with states  $P = \{p_0, p_1\}$ , stack alphabet  $\Gamma = \{a, b, c\}$ , and the following transitions:

$$\begin{aligned}
 p_0 a &\rightarrow p_0 \\
 p_0 b &\rightarrow p_1 c \\
 p_0 c &\rightarrow p_1 \\
 p_1 c &\rightarrow p_0 bb
 \end{aligned}$$

Consider the set of configurations  $C = \{p_0aac\}$ , accepted by the  $P$ -automaton below:



Apply the algorithm done in class to compute the automata  $\mathcal{A}'$  accepting  $pre^*(C)$ .

4. Describe the algorithm for computing  $post^*$  (see Fig. 4.2 on page 134 of the notes). Apply it to the PDS  $\mathcal{P}$  and set of configurations  $C$  above, to compute a  $P$ -automaton  $B$  accepting  $post^*(C)$ .
5. Prove that the intersection of a CFL and a regular language is always a CFL. Assume you are given a PDA  $\mathcal{M}$  for the CFL and an NFA  $\mathcal{A}$  for the regular language, then describe a PDA which accepts the intersection of their languages.
6. Prove that PDA's are closed under the prefix operation. That is, given a PDA  $M$  you can construct a PDA  $M'$  that accepts the prefix closure of  $L(M)$ .

Prove similarly that DPDA's are closed under prefixes.