Automata Theory and Computability

Assignment 5

Due on Fri 29 November 2013.

1. Show that the following functions are computable by a Turing Machine in the sense discussed in class. Assume that the inputs are given in unary, with 0^n representing the number n. Give a complete description of the moves of the TM using the state diagram notation.

   (a) $\mathit{square} : \mathbb{N} \to \mathbb{N}$ where $\mathit{square}(n) = n^2$.

   (b) (integer division) $\mathit{div} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, where $\mathit{div}(m, n)$ is the largest integer less than or equal to m/n if $m \geq 0$ and 0 otherwise.

2. One of the following sets is r.e. and the other is not. Which is which?

   (a) $\{M \mid L(M)$ contains at least 481 elements $\}$.

   (b) $\{M \mid L(M)$ contains at most 481 elements $\}$.

3. Let $L, K \subseteq \Sigma^*$. Define

$$L/K = \{x \mid \exists y \in K,\ xy \in L\}$$

   (a) Show that if L is regular and K is *any* language, then L/K is regular.

   (b) Show that even if we are given a DFA for L and a Turing machine for K, we cannot always construct an automaton for L/K.

4. Show without using Rice's theorem that neither the language

$$\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$$

   nor its complement is r.e.

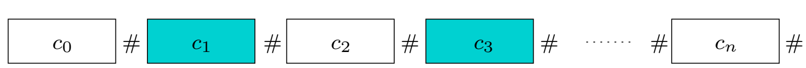
5. Is it decidable to tell for a given CFG G whether the complement of $L(G)$ is also a CFL? Justify your answer.

6. Show that it is undecidable to check whether the intersection of two DCFL's is non-empty.

   Hint: represent a valid halting computation of M on x by a string of the form

$$c_0\#\mathit{rev}(c_1)\#c_2\#\mathit{rev}(c_3)\#\cdots c_n\#,$$

   as shown below, which is similar to the encoding we used for VALCOMPS(M, x), except that each alternate configuration is *reversed*.



Show that this set of strings can be expressed as the intersection of two DCFL's.

7. Is the language inclusion problem for *deterministic* pushdown automata (DPDA's) (i.e. given DPDA's M_1 and M_2, is $L(M_1) \subseteq L(M_2)$?) decidable? Justify your answer.