

More undecidable problems

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20 November 2012

Outline

- 1 More problems about Turing Machines

More decidable/undecidable problems

Problem (a)

Is it decidable whether a given Turing machine has at least 481 states? Assume that the TM is given using the encoding below:

$$0^n 10^m 10^k 10^s 10^t 10^r 10^u 10^v \ 1 \ 0^p 10^a 10^q 10^b 10 \ 1 \ 0^{p'} 10^{a'} 10^{q'} 10^{b'} 100 \ \dots \ 1 \ 0^{p''} 10^{a''} 10^{q''} 10^{b''} 10.$$

00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

More decidable/undecidable problems

Problem (a)

Is it decidable whether a given Turing machine has at least 481 states? Assume that the TM is given using the encoding below:

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00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

Yes, it is.

We can give a TM N which given $enc(M)$

- Counts the number of states in M upto 481.
- Accepts if it reaches 481, rejects otherwise.

More decidable/undecidable problems

Problem (b)

Is it decidable whether a given Turing machine takes more than 481 steps on input ϵ without halting?

00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

More decidable/undecidable problems

Problem (b)

Is it decidable whether a given Turing machine takes more than 481 steps on input ϵ without halting?

00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

Yes, it is.

We can give a TM N which given $enc(M)$

- Uses 4 tapes: On the 4th tape it writes 481 0's.
- Uses the first 3 tapes to simulate M on input ϵ , like the universal TM U .
- Blanks out a 0 from 4th tape for each 1-step simulation done by U .
- Rejects if M halts before all 0's are blanked out on 4th tape, accepts otherwise.

More decidable/undecidable problems

Problem (c)

Is it decidable whether a given Turing machine takes more than 481 steps on *some* input without halting?

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More decidable/undecidable problems

Problem (c)

Is it decidable whether a given Turing machine takes more than 481 steps on *some* input without halting?

00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

Yes, it is.

Check if M runs for more than 481 steps on each input x of length upto 481. If so accept, else reject.

	1	2	3		481	482				
⊢	a	a	b	a	b	a	a	b	b	

More decidable/undecidable problems

Problem (d)

Is it decidable whether a given Turing machine takes more than 481 steps on *all* inputs without halting?

00010000100101001000100010000 1 010001010000100 1 0100100100100 1 010101010.

More decidable/undecidable problems

Problem (d)

Is it decidable whether a given Turing machine takes more than 481 steps on *all* inputs without halting?

00010000100101001000100010000 1 010001010000100 1 0100100100100 1 010101010.

Yes, it is.

Check if M runs for more than 481 steps on each input x of length upto 481. If so accept, else reject.

	1	2	3		481	482				
⊢	a	a	b	a	b	a	a	b	b	

More decidable/undecidable problems

Problem (e)

Is it decidable whether a given Turing machine moves its head more than 481 cells away from the left-end marker, on input ϵ ?

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More decidable/undecidable problems

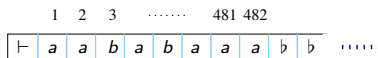
Problem (e)

Is it decidable whether a given Turing machine moves its head more than 481 cells away from the left-end marker, on input ϵ ?

00010000100101001000100010000 1 01000101000100 1 0100100100100 1 010101010.

Yes, it is.

Simulate M on ϵ for upto $m^{481} \cdot 482 \cdot k$ steps. If M visits the 482nd cell, accept, else reject.



More decidable/undecidable problems

Problem (f)

Is it decidable whether a given Turing machine accepts the null-string ϵ ?

More decidable/undecidable problems

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Is it decidable whether a given Turing machine accepts the null-string ϵ ?

No.

If it were decidable, say by a TM N , then we could use N to decide HP as follows: Define a new machine N' which given input $M\#x$, outputs the description of a machine M' which:

- erases its input
- writes x on its input tape
- Behaves like M on x
- Accepts if M halts on x .

N' then calls N with input M' .

N accepts M' iff M' accepts ϵ iff M halts on x .

Turing machine M' for Problem (f)

$$L(M') = \begin{cases} A^* & \text{if } M \text{ halts on } x \\ \emptyset & \text{if } M \text{ does not halt on } x. \end{cases}$$

More decidable/undecidable problems

Problem (g)

Is it decidable whether a given Turing machine accepts any string at all? That is, is $L(M) \neq \emptyset$?

More decidable/undecidable problems

Problem (h)

Is it decidable whether a given Turing machine accepts all strings?
That is, is $L(M) = A^*$?

More decidable/undecidable problems

Problem (i)

Is it decidable whether a given Turing machine accepts a finite set?

More decidable/undecidable problems

Problem (j)

Is it decidable whether a given Turing machine accepts a regular set?

More decidable/undecidable problems

Problem (j)

Is it decidable whether a given Turing machine accepts a regular set?

Given M and x , build a new machine M' that behaves as follows:

- ① Saves its input y on tape 2.
- ② writes x on tape 1.
- ③ runs as M on x .
- ④ if M gets into a halting state, then
 - M' takes back control,
 - Runs as M_R on y ,
 - (Here M_R is any TM that accepts a non-regular language R , say $R = \{a^n b^n \mid n \geq 0\}$).
 - M' accepts iff M_R accepts.

Turing machine M' for Problem (j)

$$L(M') = \begin{cases} R & \text{if } M \text{ halts on } x \\ \emptyset & \text{if } M \text{ does not halt on } x. \end{cases}$$

More decidable/undecidable problems

Problem (k)

Is it decidable whether a given Turing machine accepts a CFL?

More decidable/undecidable problems

Problem (I)

Is it decidable whether a given Turing machine accepts a recursive set?