Applications of Context Free Grammar in Natural Language Processing

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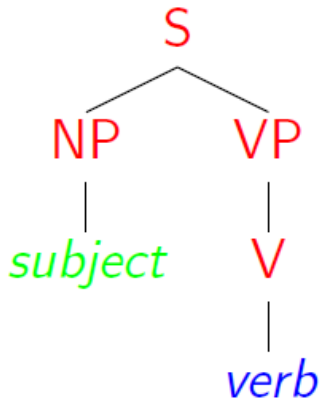
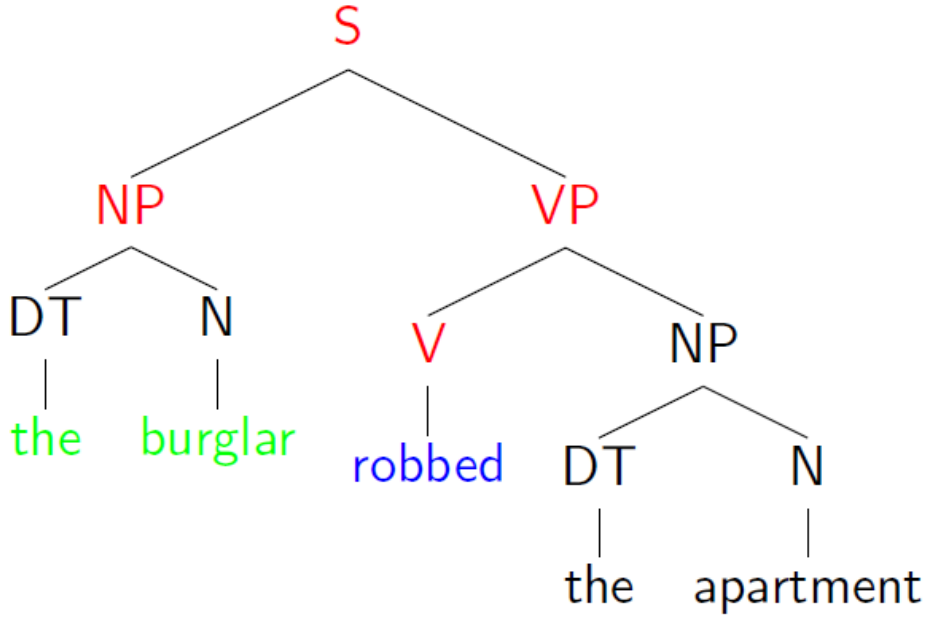
(Based on slides of Michael Collins, Dan Jurafsky, Dan Klein, Chris Manning, Ray Mooney, Luke Zettlemoyer)

Why Parse?

- Part of speech information
- Phrase information
- Useful relationships





$\Rightarrow$ "the burglar" is the subject of "robbed"

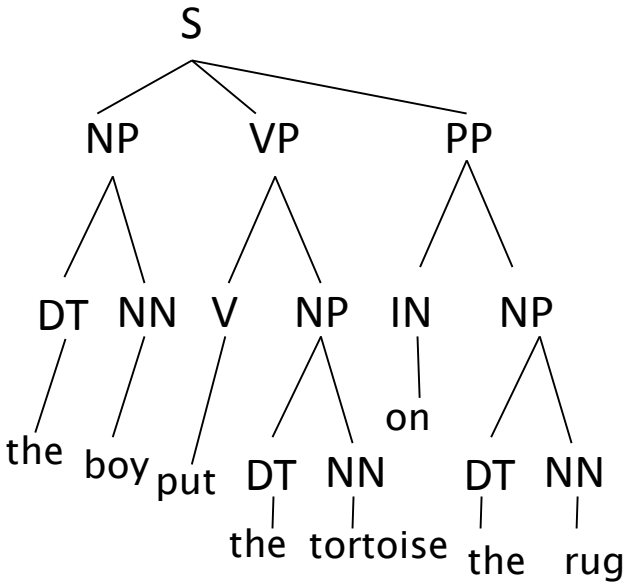
Statistical parsing applications

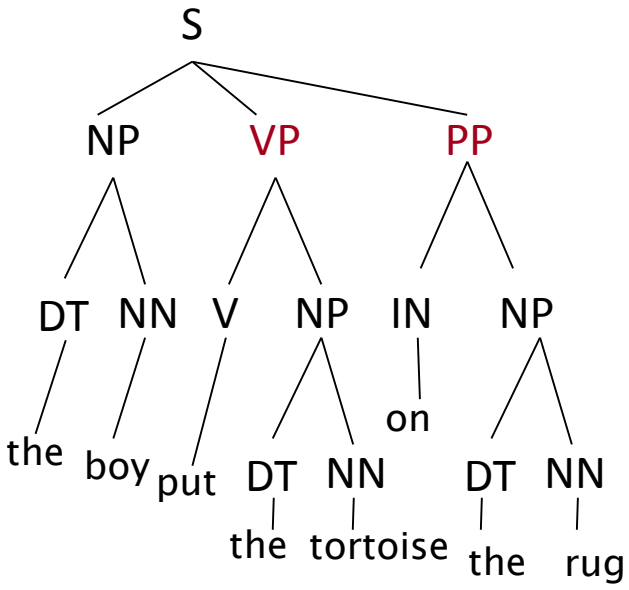
Statistical parsers are now robust and widely used in larger NLP applications:

- High precision question answering [Pasca and Harabagiu SIGIR 2001]
- Improving biological named entity finding [Finkel et al. JNLPBA 2004]
- Syntactically based sentence compression [Lin and Wilbur 2007]
- Extracting opinions about products [Bloom et al. NAACL 2007]
- Improved interaction in computer games [Gorniak and Roy 2005]
- Helping linguists find data [Resnik et al. BLS 2005]
- Source sentence analysis for machine translation [Xu et al. 2009]
- Relation extraction systems [Fundel et al. *Bioinformatics* 2006]

Example Application: Machine Translation

- The boy put the tortoise on the rug
- लड़के ने रखा कछुआ ऊपर कालीन
- SVO vs. SOV; preposition vs. post-position





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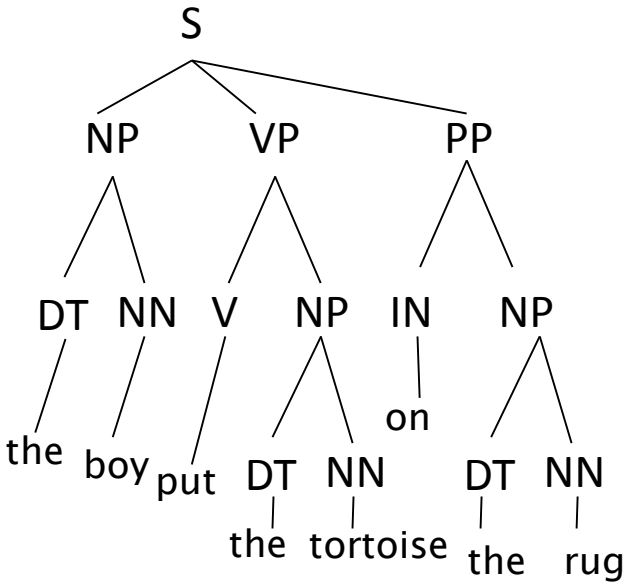

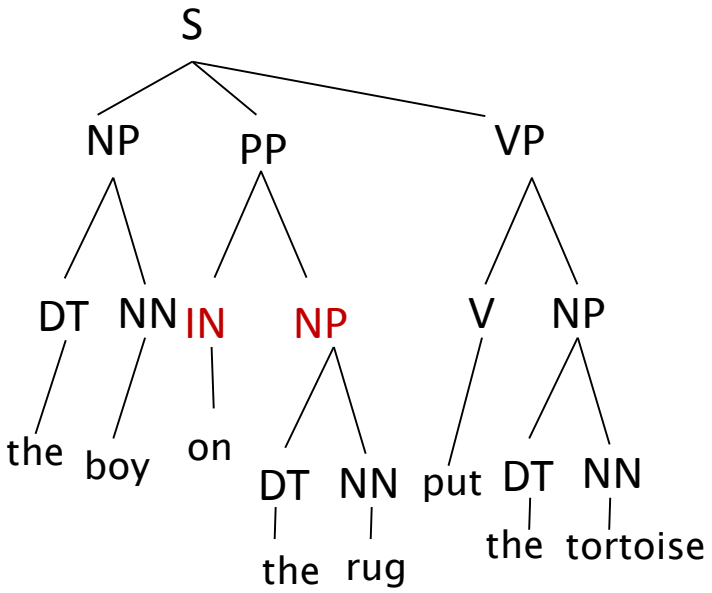

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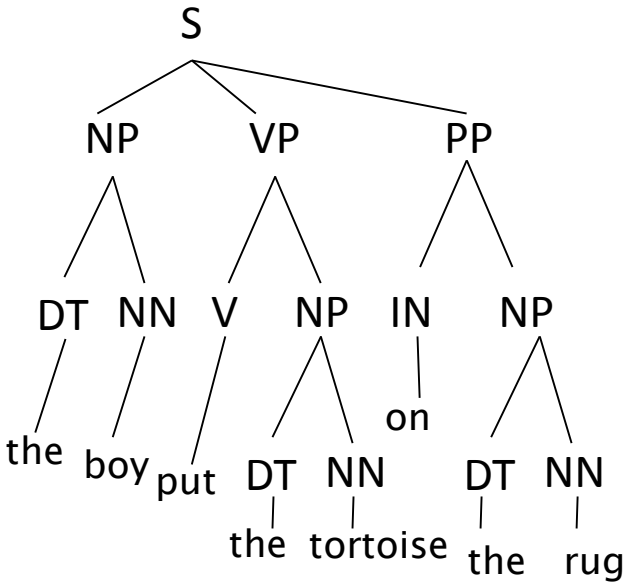

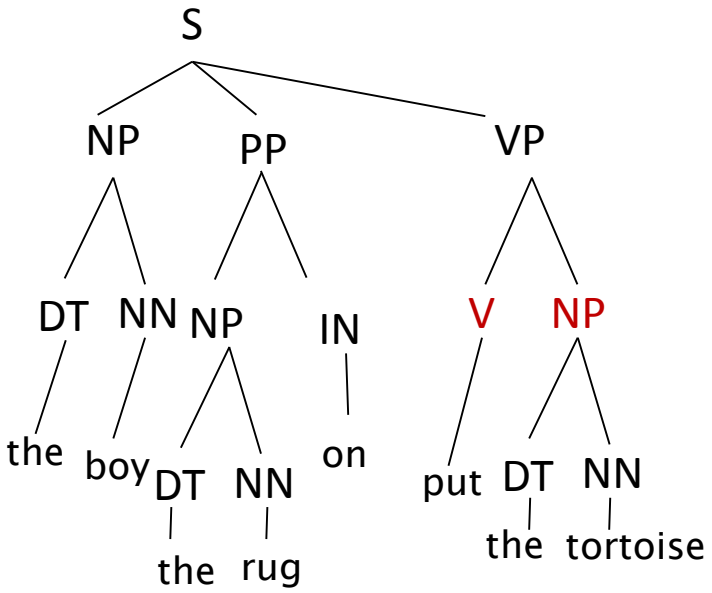

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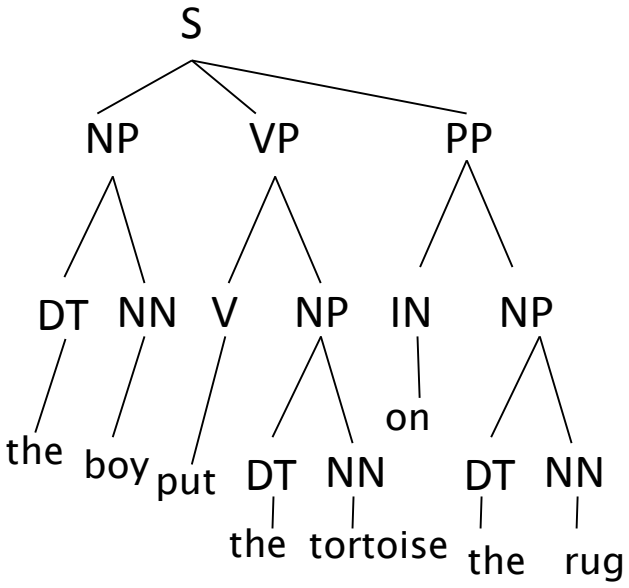

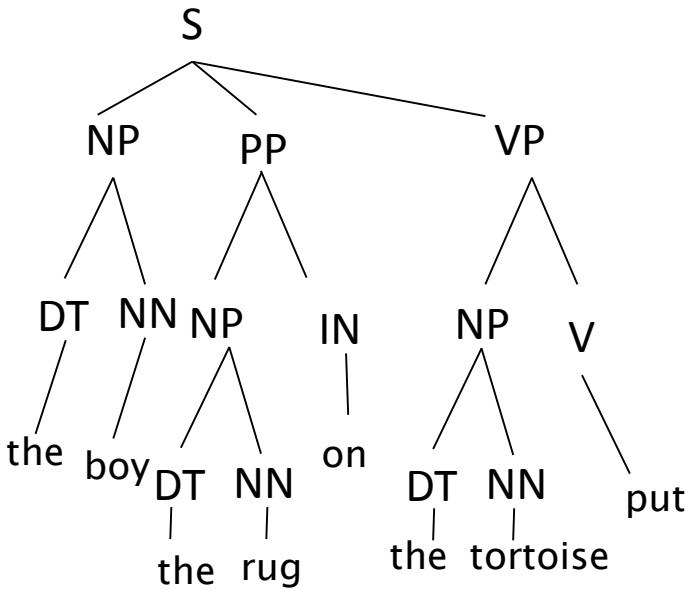

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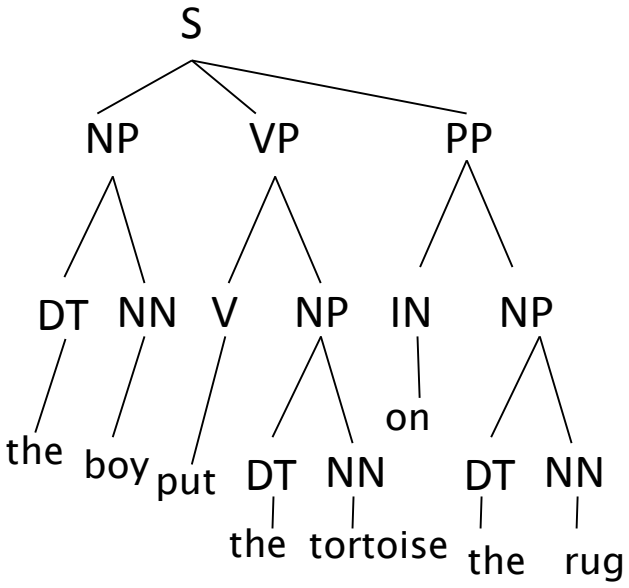
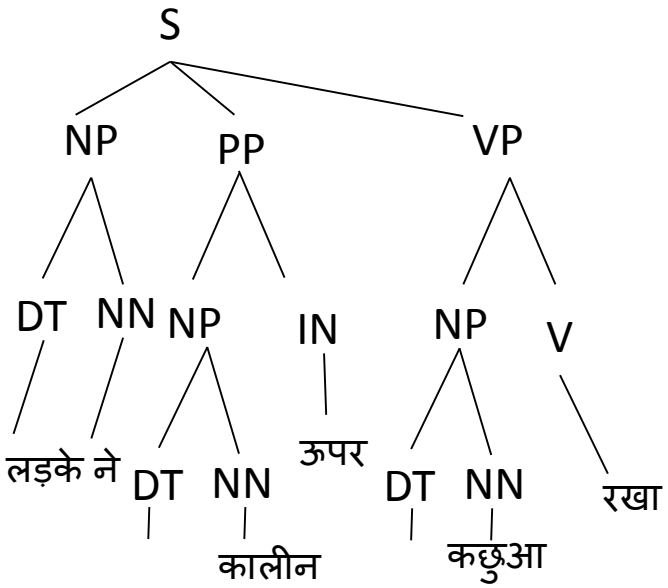




Context Free Grammars and Ambiguities

Context-Free Grammars in NLP

- A context free grammar G in NLP = (N, C, Σ, S, L, R)
  - Σ is a set of terminal symbols
  - C is a set of preterminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol (S ∈ N)
  - L is the lexicon, a set of items of the form $X \rightarrow x$
    - X ∈ C and x ∈ Σ
  - R is the grammar, a set of items of the form $X \rightarrow \gamma$
    - X ∈ N and $\gamma \in (N \cup C)^*$
- By usual convention, S is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write *e* for an empty sequence, rather than nothing

A Context Free Grammar of English

$N = \{\text{S, NP, VP, PP, DT, Vi, Vt, NN, IN}\}$

$S = \text{S}$

$\Sigma = \{\text{sleeps, saw, man, woman, telescope, the, with, in}\}$

$R =$

S	$\rightarrow$	NP	VP
VP	$\rightarrow$	Vi	
VP	$\rightarrow$	Vt	NP
VP	$\rightarrow$	VP	PP
NP	$\rightarrow$	DT	NN
NP	$\rightarrow$	NP	PP
PP	$\rightarrow$	IN	NP

Vi	$\rightarrow$	sleeps
Vt	$\rightarrow$	saw
NN	$\rightarrow$	man
NN	$\rightarrow$	woman
NN	$\rightarrow$	telescope
DT	$\rightarrow$	the
IN	$\rightarrow$	with
IN	$\rightarrow$	in

Note: S=sentence, VP=verb phrase, NP=noun phrase, PP=prepositional phrase, DT=determiner, Vi=intransitive verb, Vt=transitive verb, NN=noun, IN=preposition

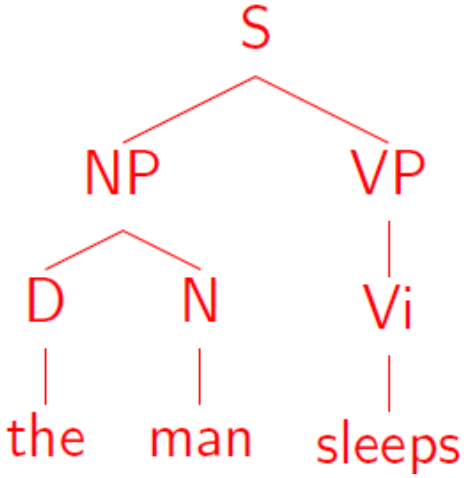
Left-Most Derivations

A left-most derivation is a sequence of strings $s_1 \ldots s_n$, where

- $s_1 = S$, the start symbol
- $s_n \in \Sigma^*$, i.e. s_n is made up of terminal symbols only
- Each s_i for $i = 2 \ldots n$ is derived from s_{i-1} by picking the left-most non-terminal X in s_{i-1} and replacing it by some β where $X \rightarrow \beta$ is a rule in R

For example: [S], [NP VP], [D N VP], [the N VP], [the man VP], [the man Vi], [the man sleeps]

Representation of a derivation as a tree:



Properties of CFGs

- A CFG defines a set of possible derivations
- A string $s \in \Sigma^*$ is in the *language* defined by the CFG if there is at least one derivation that yields s
- Each string in the language generated by the CFG may have more than one derivation ("ambiguity")

Attachment ambiguities

- A key parsing decision is how we ‘attach’ various constituents
  - PPs, adverbial or participial phrases, infinitives, coordinations, etc.

The board approved [its acquisition] [by Royal Trustco Ltd.]
[of Toronto]
[for $27 a share]
[at its monthly meeting].

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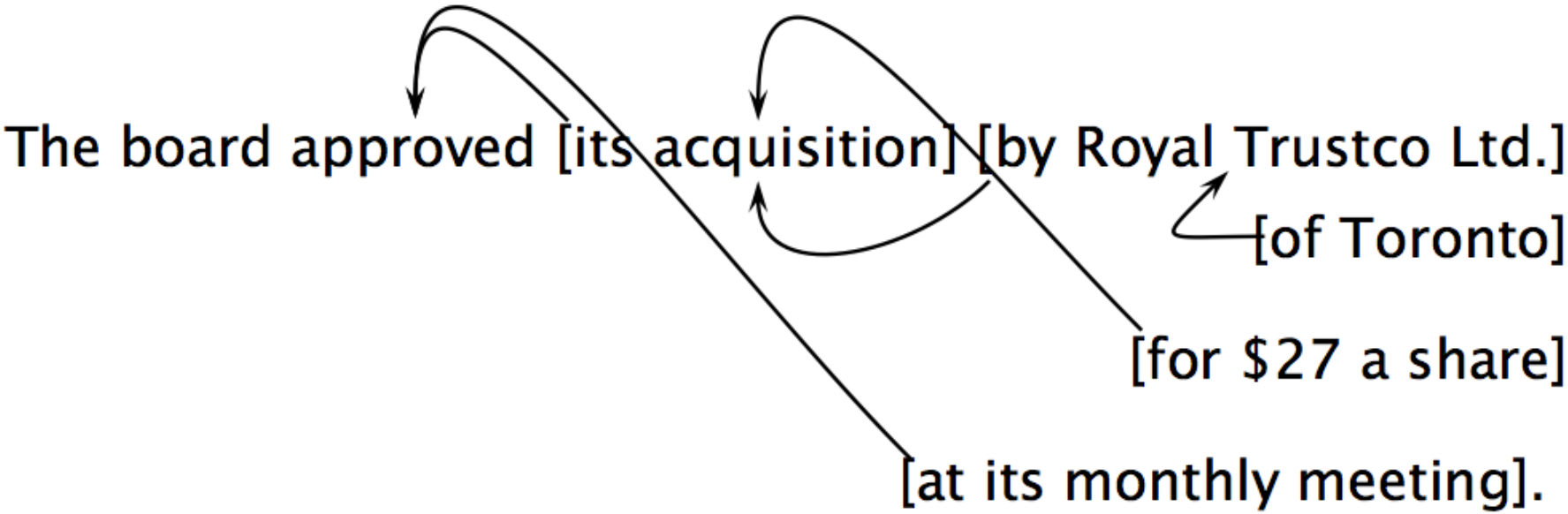
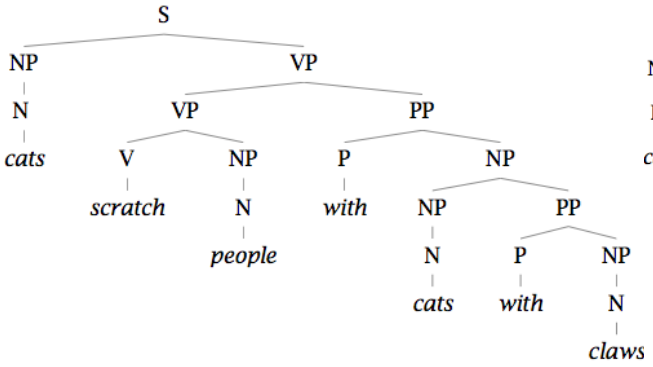
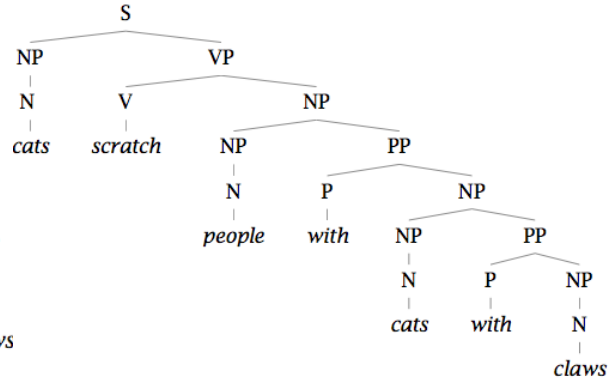
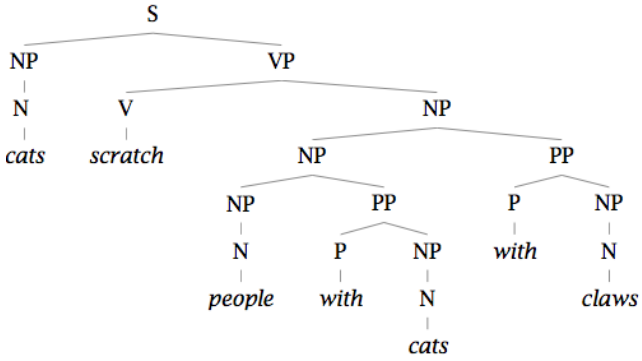


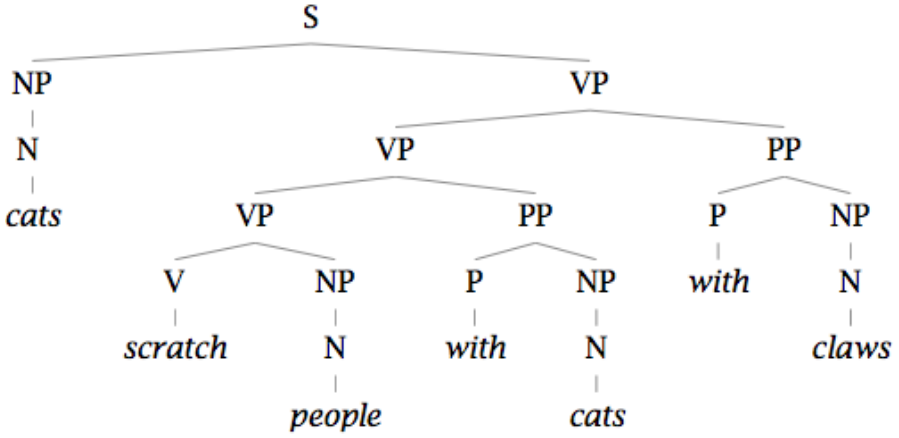
- Catalan numbers: $C_n = (2n)!/[(n+1)!n!]$
- An exponentially growing series, which arises in many tree-like contexts:
  - E.g., the number of possible triangulations of a polygon with $n+2$ sides
    - Turns up in triangulation of probabilistic graphical models….

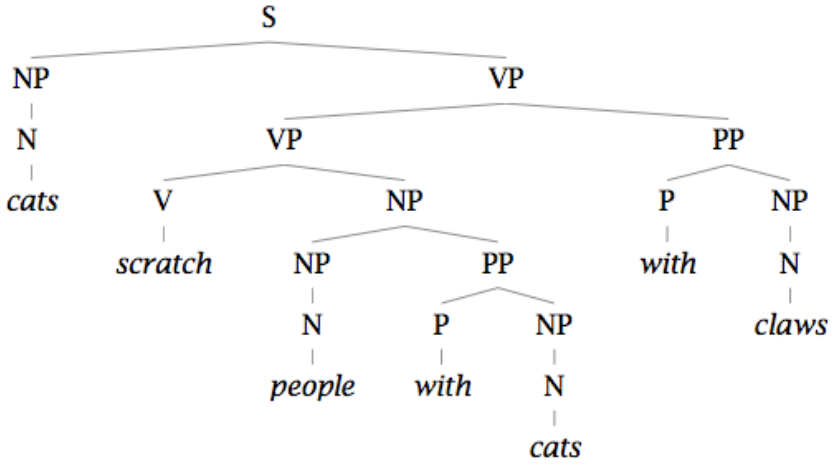
Parsing: Two problems to solve:
1. Repeated work…



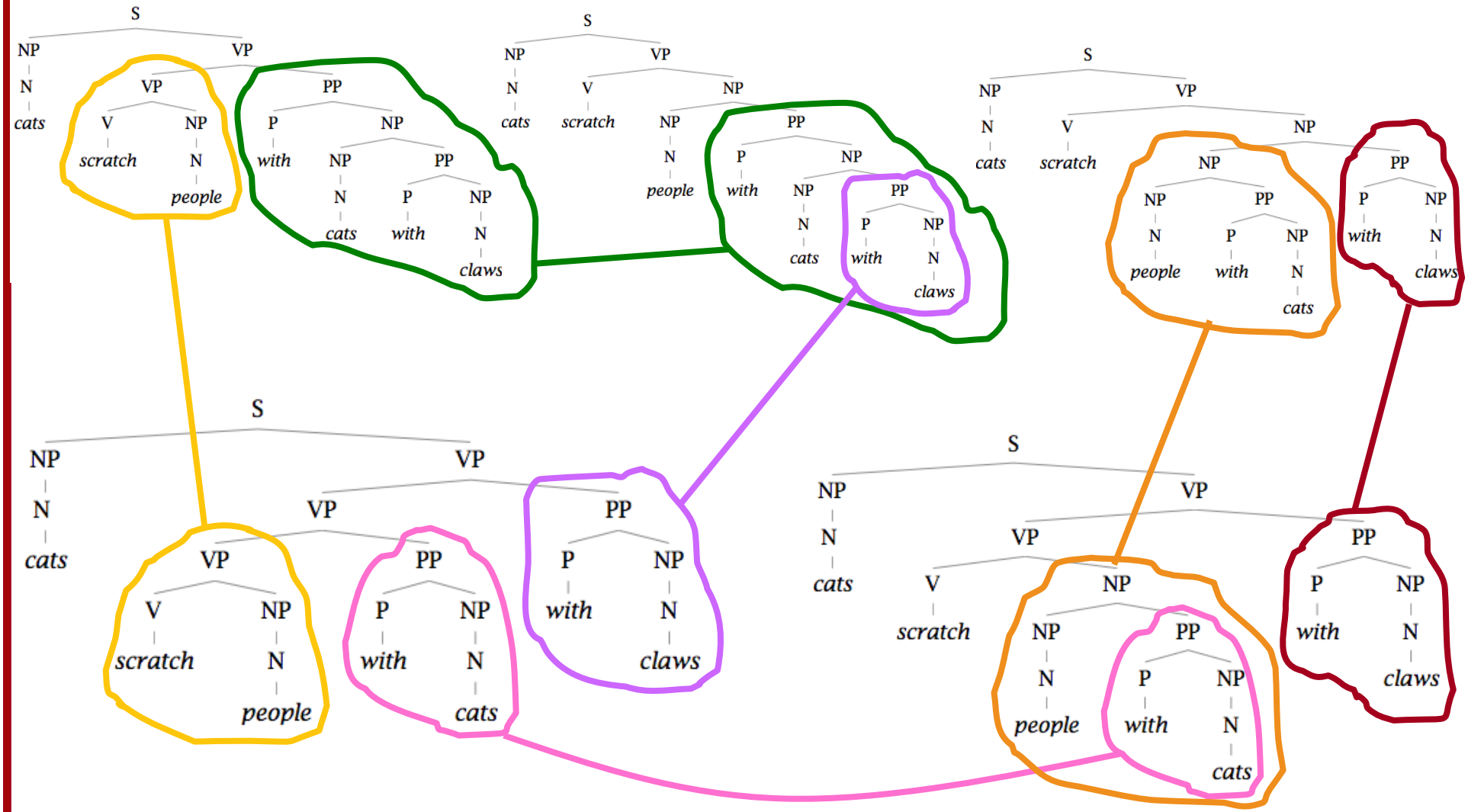








Parsing: Two problems to solve:
1. Repeated work…



Parsing: Two problems to solve:
2. Choosing the correct parse

- How do we work out the correct attachment:
  - She saw the man with a telescope
- Is the problem 'AI complete'? Yes, but …
- Words are good predictors of attachment
  - Even absent full understanding
  - Moscow sent more than 100,000 soldiers into Afghanistan …
  - Sydney Water breached an agreement with NSW Health …
- Our statistical parsers will try to exploit such statistics.

Classical NLP Parsing: The problem and its solution

- Categorical constraints can be added to grammars to limit unlikely/weird parses for sentences
  - But the attempt make the grammars not robust
    - In traditional systems, commonly 30% of sentences in even an edited text would have *no* parse.
- A less constrained grammar can parse more sentences
  - But simple sentences end up with ever more parses with no way to choose between them
- We need mechanisms that allow us to find ***the most likely parse(s)*** for a sentence
  - Statistical parsing lets us work with very loose grammars that admit millions of parses for sentences but still quickly find the best parse(s)

The rise of annotated data: The Penn Treebank

[Marcus et al. 1993, *Computational Linguistics*]

```
( (S
    (NP-SBJ (DT The) (NN move))
    (VP (VBD followed)
      (NP
        (NP (DT a) (NN round))
        (PP (IN of)
          (NP
            (NP (JJ similar) (NNS increases))
            (PP (IN by)
              (NP (JJ other) (NNS lenders)))
            (PP (IN against)
              (NP (NNP Arizona) (JJ real) (NN estate) (NNS loans))))))
      (, ,)
      (S-ADV
        (NP-SBJ (-NONE- *))
        (VP (VBG reflecting)
          (NP
            (NP (DT a) (VBG continuing) (NN decline))
            (PP-LOC (IN in)
              (NP (DT that) (NN market)))))))
    (. .)))
```

Penn Treebank Non-terminals

Table 1.2. The Penn Treebank syntactic tagset

ADJP	Adjective phrase
ADVP	Adverb phrase
NP	Noun phrase
PP	Prepositional phrase
S	Simple declarative clause
SBAR	Subordinate clause
SBARQ	Direct question introduced by *wh*-element
SINV	Declarative sentence with subject-aux inversion
SQ	Yes/no questions and subconstituent of SBARQ excluding *wh*-element
VP	Verb phrase
WHADVP	Wh-adverb phrase
WHNP	Wh-noun phrase
WHPP	Wh-prepositional phrase
X	Constituent of unknown or uncertain category
*	"Understood" subject of infinitive or imperative
0	Zero variant of *that* in subordinate clauses
T	Trace of wh-Constituent

The rise of annotated data

- Starting off, building a treebank seems a lot slower and less useful than building a grammar

- But a treebank gives us many things
  - Reusability of the labor
    - Many parsers, POS taggers, etc.
    - Valuable resource for linguistics
  - Broad coverage
  - Frequencies and distributional information
  - A way to evaluate systems

Probabilistic Context Free Grammar

Probabilistic – or stochastic – context-free grammars (PCFGs)

- G = (Σ, N, S, R, P)
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol (S ∈ N)
  - R is a set of rules/productions of the form $X \rightarrow \gamma$
  - P is a probability function
    - P: R $\rightarrow$ [0,1]
    - $\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$
- A grammar G generates a language model L.

$$\sum_{\gamma \in T^*} P(\gamma) = 1$$

PCFG Example

S	⇒	NP	VP	1.0
VP	⇒	Vi		0.4
VP	⇒	Vt	NP	0.4
VP	⇒	VP	PP	0.2
NP	⇒	DT	NN	0.3
NP	⇒	NP	PP	0.7
PP	⇒	P	NP	1.0

Vi	⇒	sleeps	1.0
Vt	⇒	saw	1.0
NN	⇒	man	0.7
NN	⇒	woman	0.2
NN	⇒	telescope	0.1
DT	⇒	the	1.0
IN	⇒	with	0.5
IN	⇒	in	0.5

- Probability of a tree t with rules

$$\alpha_1 \rightarrow \beta_1, \alpha_2 \rightarrow \beta_2, \ldots, \alpha_n \rightarrow \beta_n$$

is

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$$

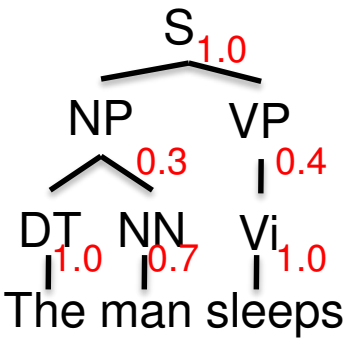
where $q(\alpha \rightarrow \beta)$ is the probability for rule $\alpha \rightarrow \beta$.

Probability of a Parse

S	⇒	NP	VP	1.0
VP	⇒	Vi		0.4
VP	⇒	Vt	NP	0.4
VP	⇒	VP	PP	0.2
NP	⇒	DT	NN	0.3
NP	⇒	NP	PP	0.7
PP	⇒	P	NP	1.0

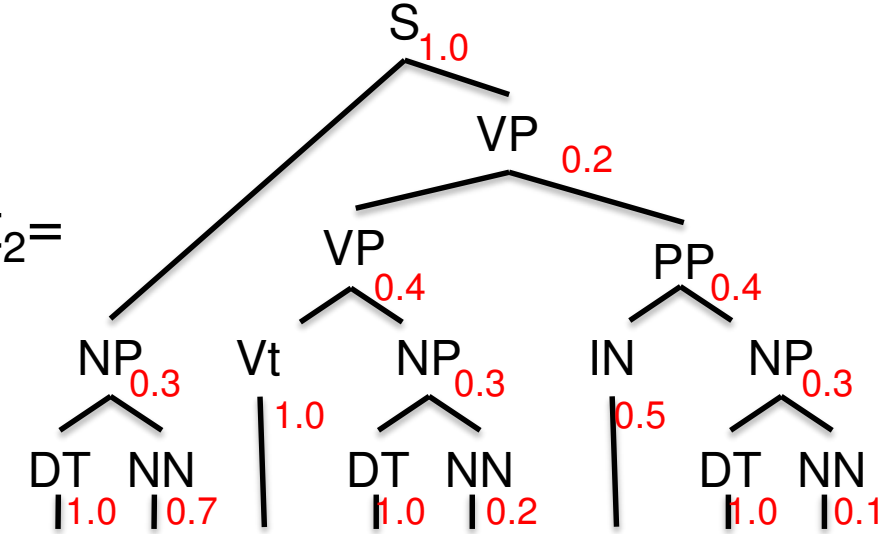
Vi	⇒	sleeps	1.0
Vt	⇒	saw	1.0
NN	⇒	man	0.7
NN	⇒	woman	0.2
NN	⇒	telescope	0.1
DT	⇒	the	1.0
IN	⇒	with	0.5
IN	⇒	in	0.5

t_1=



$p(t_1)=1.0*0.3*1.0*0.7*0.4*1.0$

t_2=



$p(t_s)=1.8*0.3*1.0*0.7*0.2*0.4*1.0*0.3*1.0*0.2*0.4*0.5*0.3*1.0*0.1$

PCFGs: Learning and Inference

- Model
  - The probability of a tree t with n rules $\alpha_i \rightarrow \beta_i$, i = 1..n

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$$

- Learning
  - Read the rules off of labeled sentences, use ML estimates for probabilities

$$q_{ML}(\alpha \rightarrow \beta) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$$

  - and use all of our standard smoothing tricks!

- Inference
  - For input sentence s, define T(s) to be the set of trees whose *yield* is s (whole leaves, read left to right, match the words in s)

$$t^*(s) = \arg \max_{t \in \mathcal{T}(s)} p(t)$$

Grammar Transforms

Chomsky Normal Form

- All rules are of the form $X \rightarrow Y\ Z$ or $X \rightarrow w$
  - $X, Y, Z \in N$ and $w \in \Sigma$
- A transformation to this form doesn't change the weak generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- n-ary rules are divided by introducing new nonterminals ($n > 2$)

A phrase structure grammar

S → NP VP

VP → V NP

VP → V NP PP

NP → NP NP

NP → NP PP

NP → N

NP → *e*

PP → P NP

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

S → VP

VP → V NP

VP → V

VP → V NP PP

VP → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Chomsky Normal Form steps

$S \rightarrow NP\ VP$

$VP \rightarrow V\ NP$

$S \rightarrow V\ NP$

$VP \rightarrow V$

$S \rightarrow V$

$VP \rightarrow V\ NP\ PP$

$S \rightarrow V\ NP\ PP$

$VP \rightarrow V\ PP$

$S \rightarrow V\ PP$

$NP \rightarrow NP\ NP$

$NP \rightarrow NP$

$NP \rightarrow NP\ PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P\ NP$

$PP \rightarrow P$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

Chomsky Normal Form steps

S → NP VP
VP → V NP
S → V NP
VP → V
VP → V NP PP
S → V NP PP
VP → V PP
S → V PP
NP → NP NP
NP → NP
NP → NP PP
NP → PP
NP → N
PP → P NP
PP → P

N → *people*
N → *fish*
N → *tanks*
N → *rods*
V → *people*
S → *people*
V → *fish*
S → *fish*
V → *tanks*
S → *tanks*
P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V NP PP

S → V NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V @VP_V

@VP_V → NP PP

S → V @S_V

@S_V → NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

A phrase structure grammar

S → NP VP

VP → V NP

VP → V NP PP

NP → NP NP

NP → NP PP

NP → N

NP → *e*

PP → P NP

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

Chomsky Normal Form steps

S → NP VP

VP → V NP

S → V NP

VP → V @VP_V

@VP_V → NP PP

S → V @S_V

@S_V → NP PP

VP → V PP

S → V PP

NP → NP NP

NP → NP PP

NP → P NP

PP → P NP

NP → *people*

NP → *fish*

NP → *tanks*

NP → *rods*

V → *people*

S → *people*

VP → *people*

V → *fish*

S → *fish*

VP → *fish*

V → *tanks*

S → *tanks*

VP → *tanks*

P → *with*

PP → *with*

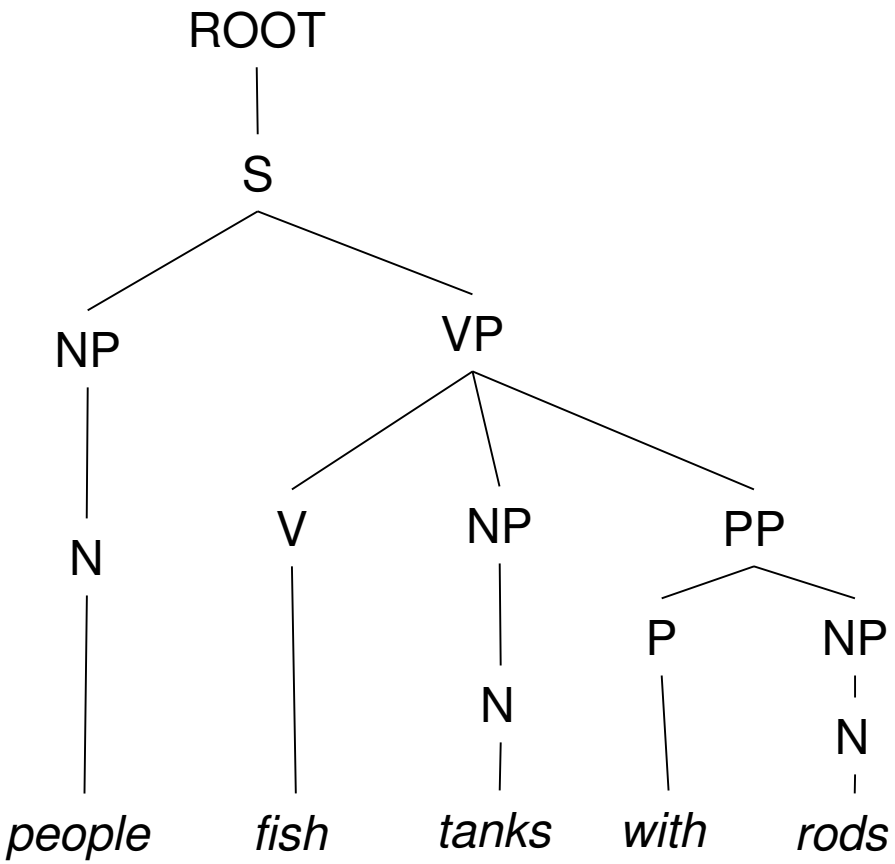
Chomsky Normal Form

- You should think of this as a transformation for efficient parsing
- With some extra book-keeping in symbol names, you can even reconstruct the same trees with a detransform
- In practice full Chomsky Normal Form is a pain
  - Reconstructing n-aries is easy
  - Reconstructing unaries/empties is trickier

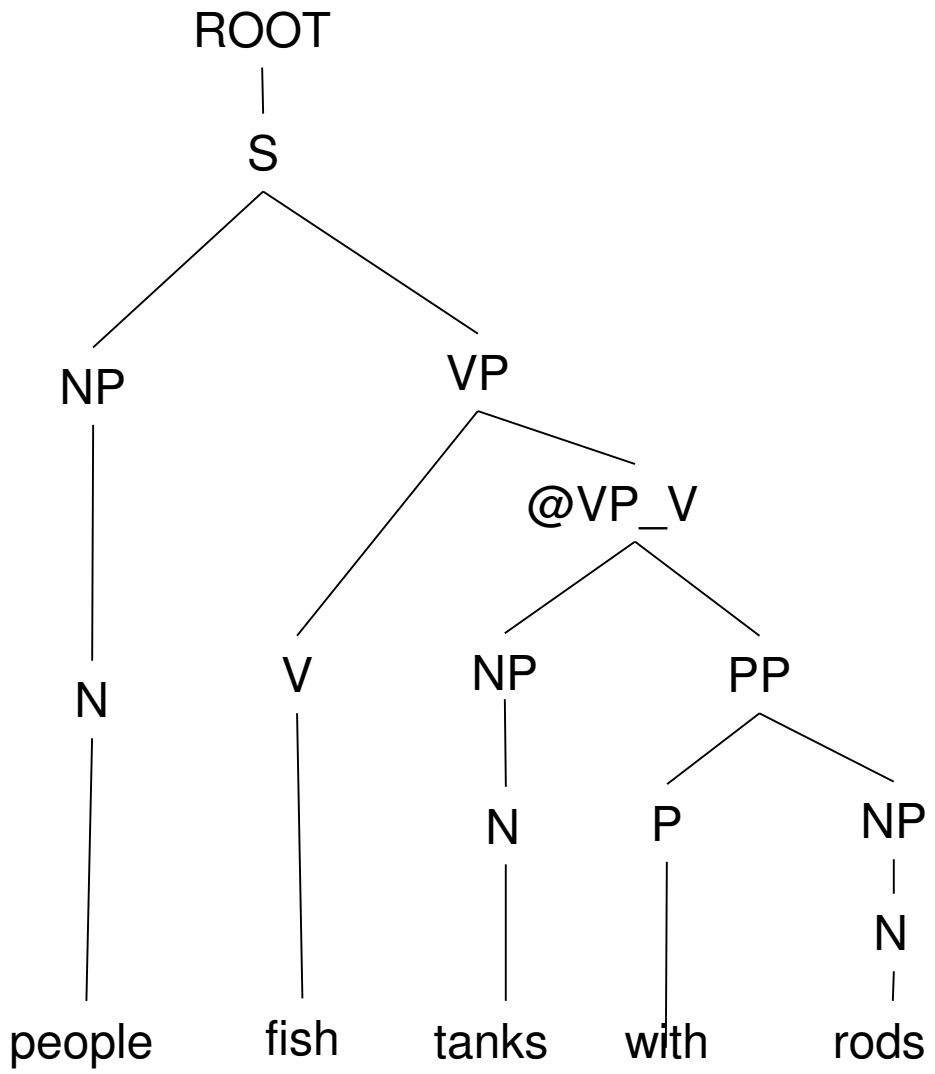
- **Binarization** is crucial for cubic time CFG parsing

- The rest isn't necessary; it just makes the algorithms cleaner and a bit quicker

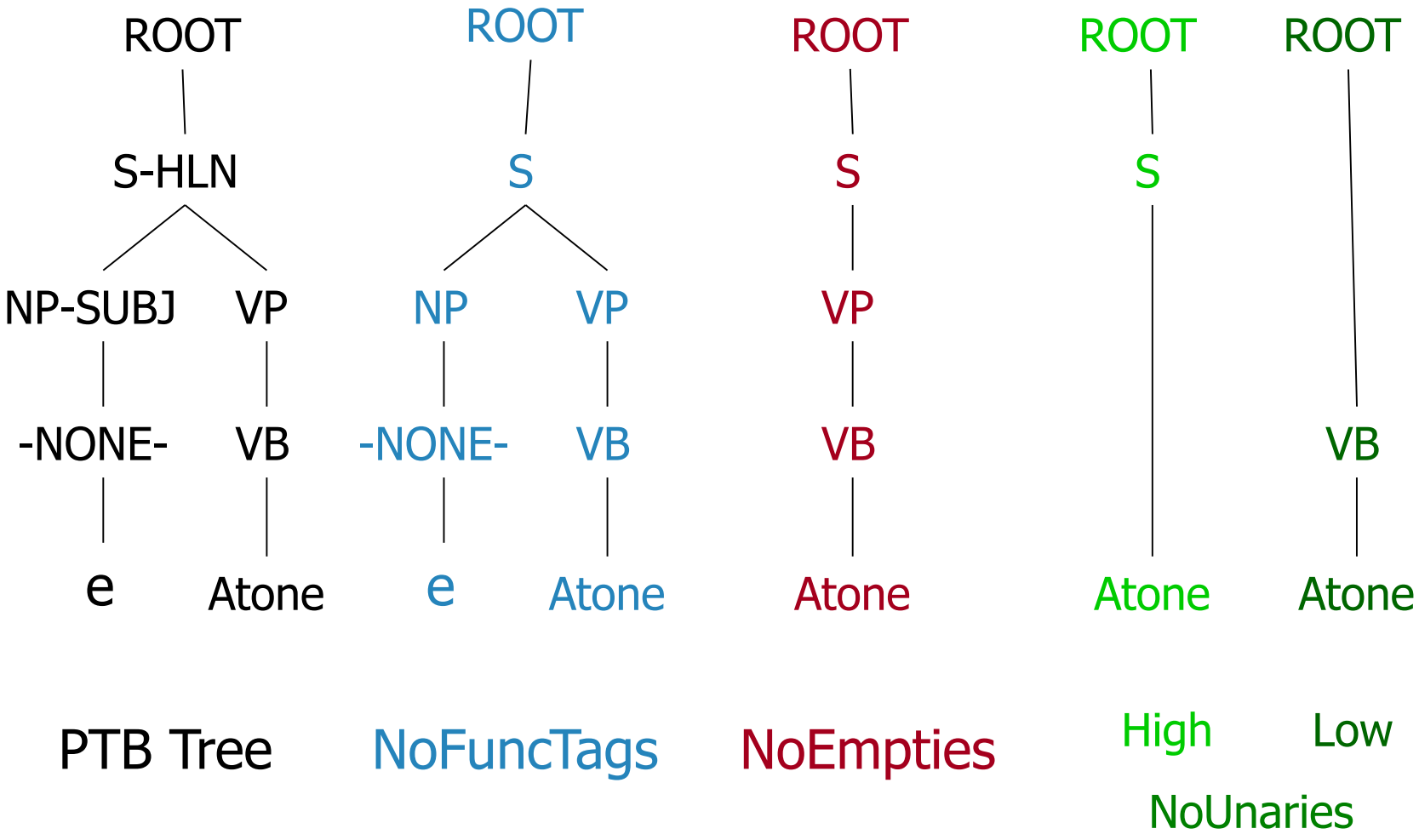
An example: before binarization…



After binarization…

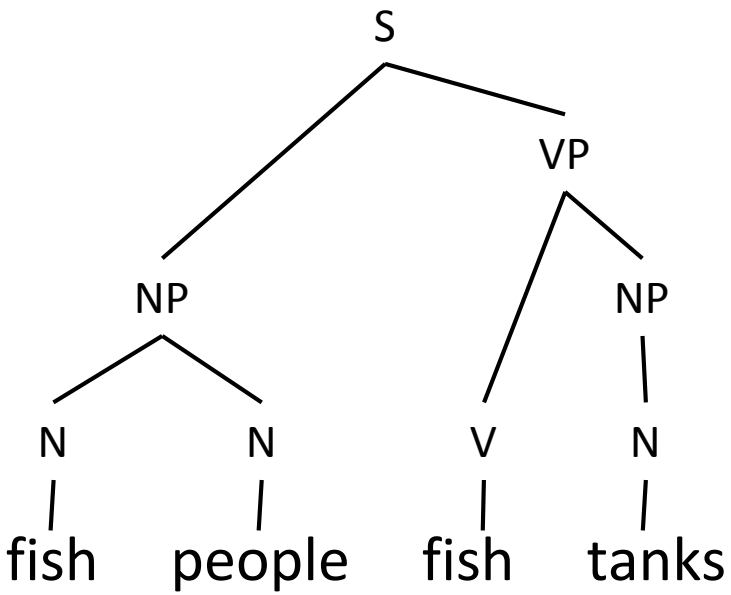


Treebank: empties and unaries



Parsing

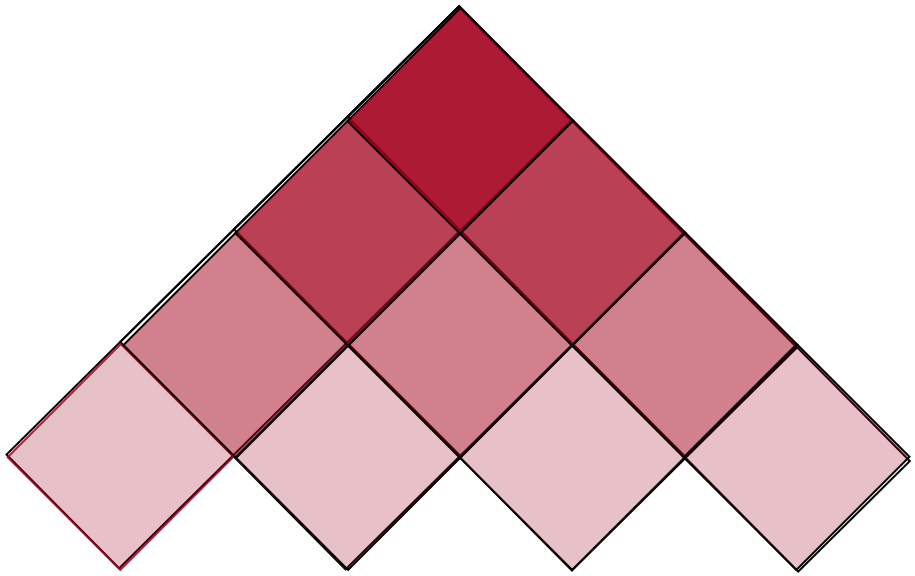
Constituency Parsing



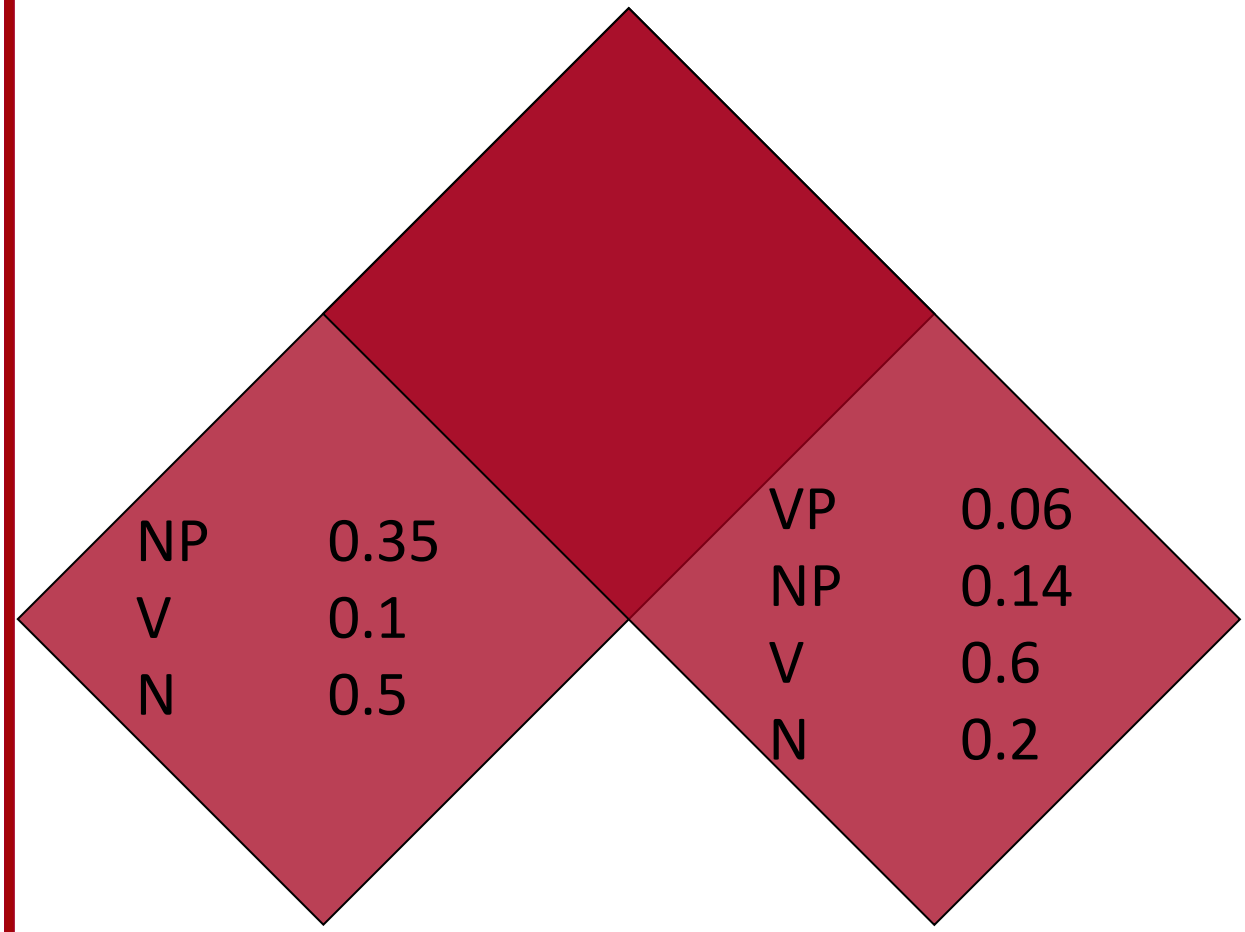
PCFG

Rule	**Prob θ_i**
$S \rightarrow NP\ VP$	θ_0
$NP \rightarrow NP\ NP$	θ_1
…	
$N \rightarrow fish$	θ_{42}
$N \rightarrow people$	θ_{43}
$V \rightarrow fish$	θ_{44}
…	

Cocke-Kasami-Younger (CKY) Constituency Parsing



Viterbi (Max) Scores



S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts
  - Doesn't increase complexity; essentially like unaries

- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar
    - Binarization may be an explicit transformation or implicit in how the parser works (Early-style dotted rules), but it's always there.

A Recursive Parser

```
bestScore(X,i,j,s)
 if (j == i)
     return q(X->s[i])
 else
     return max  q(X->YZ) *
            k,X->YZ
                   bestScore(Y,i,k,s) *
                   bestScore(Z,k+1,j,s)
```

The CKY algorithm (1960/1965)
… extended to unaries

```
function CKY(words, grammar) returns [most_probable_parse,prob]
  score = new double[#(words)+1][#(words)+1][#(nonterms)]
  back = new Pair[#(words)+1][#(words)+1][#nonterms]]
  for i=0; i<#(words); i++
    for A in nonterms
      if A -> words[i] in grammar
        score[i][i+1][A] = P(A -> words[i])
    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
          prob = P(A->B)*score[i][i+1][B]
          if prob > score[i][i+1][A]
            score[i][i+1][A] = prob
            back[i][i+1][A] = B
            added = true
```

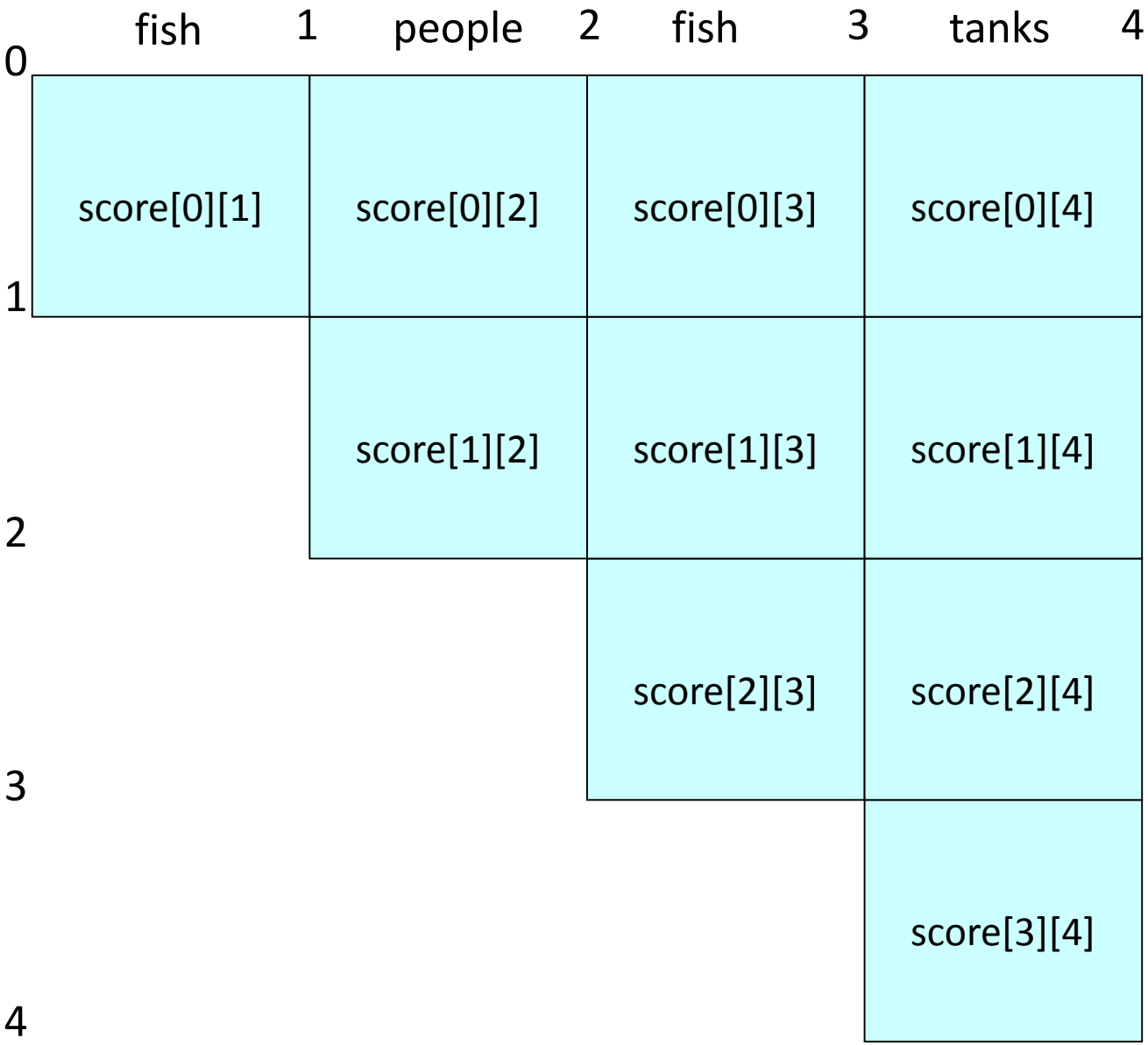
The CKY algorithm (1960/1965)
… extended to unaries

```
for span = 2 to #(words)
  for begin = 0 to #(words)- span
    end = begin + span
    for split = begin+1 to end-1
      for A,B,C in nonterms
        prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
        if prob > score[begin][end][A]
          score[begin]end][A] = prob
          back[begin][end][A] = new Triple(split,B,C)
    //handle unaries
    boolean added = true
    while added
      added = false
      for A, B in nonterms
        prob = P(A->B)*score[begin][end][B];
        if prob > score[begin][end][A]
          score[begin][end][A] = prob
          back[begin][end][A] = B
          added = true
return buildTree(score, back)
```

The grammar:
Binary, no epsilons,

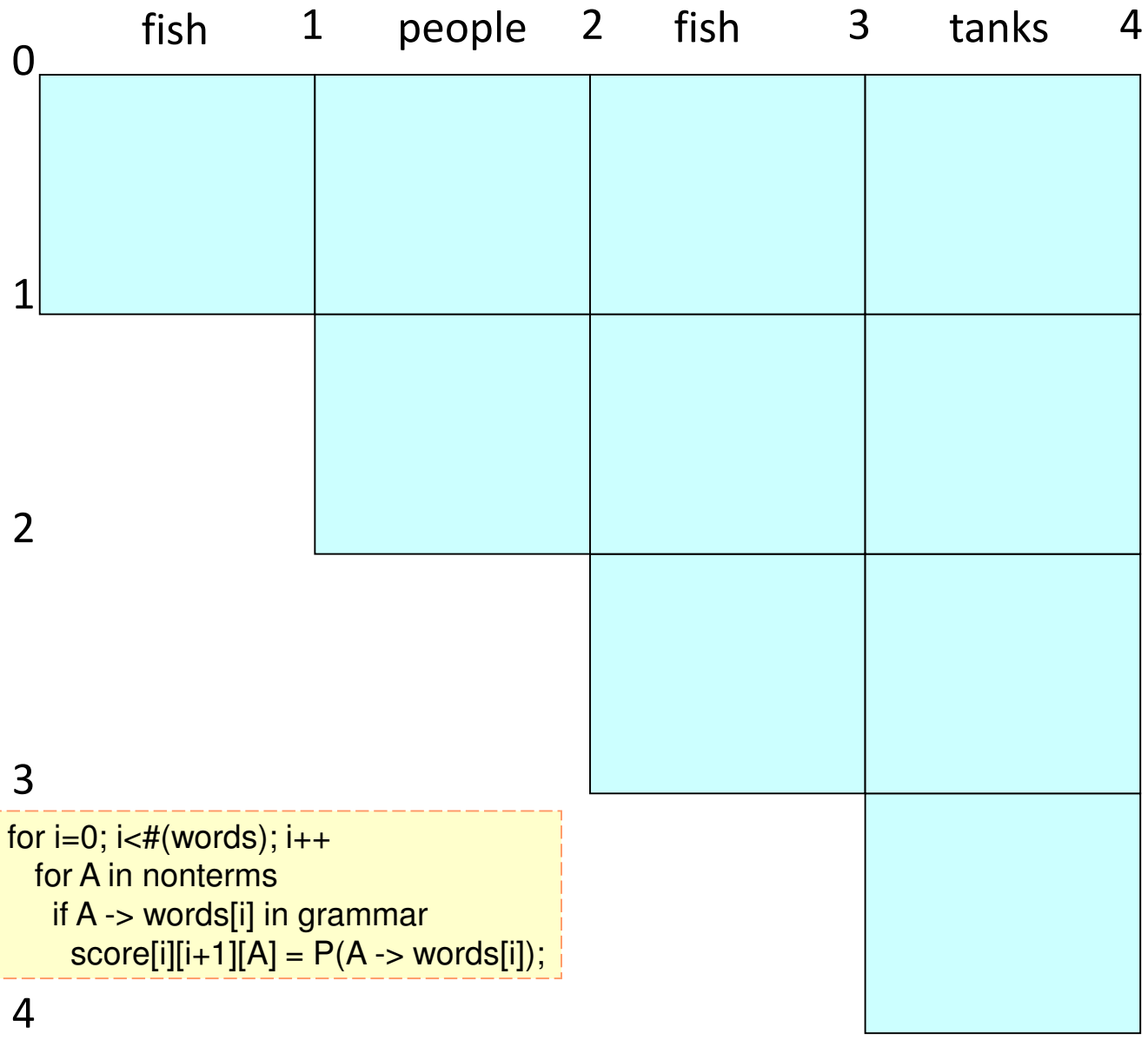
S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0


fish
1
people
2
fish
3
tanks
4
0
1
2
3
4
score[0][1]
score[0][2]
score[0][3]
score[0][4]
score[1][2]
score[1][3]
score[1][4]
score[2][3]
score[2][4]
score[3][4]

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

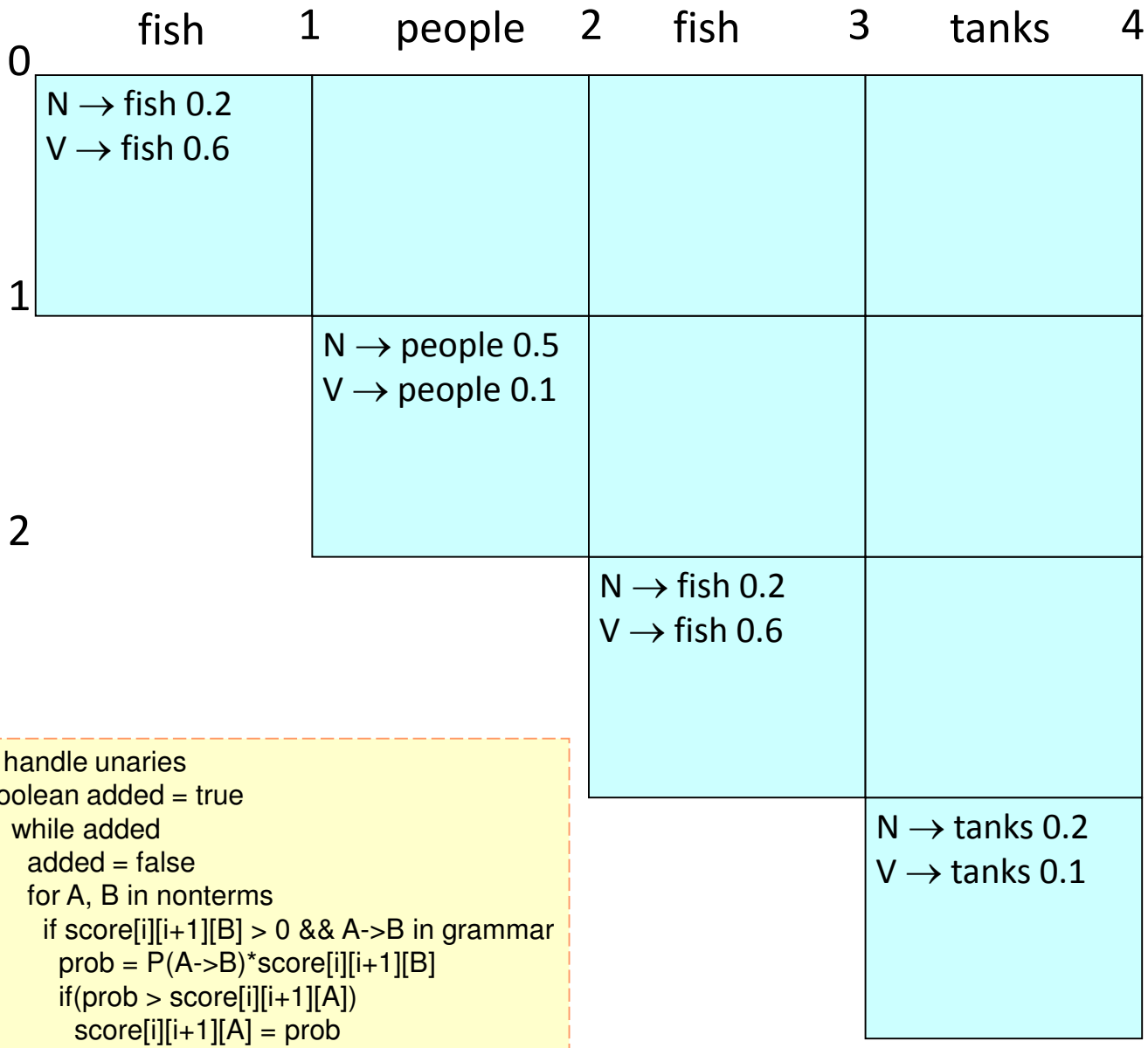
N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0




```
for i=0; i<#(words); i++
   for A in nonterms
      if A -> words[i] in grammar
         score[i][i+1][A] = P(A -> words[i]);
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0




```
// handle unaries
boolean added = true
    while added
      added = false
      for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
          prob = P(A->B)*score[i][i+1][B]
          if(prob > score[i][i+1][A])
            score[i][i+1][A] = prob
            back[i][i+1][A] = B
            added = true
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish	1	people	2	fish	3	tanks	4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006							
1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001					
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006			
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

```
prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
if (prob > score[begin][end][A])
    score[begin]end][A] = prob
    back[begin][end][A] = new Triple(split,B,C)
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish	1	people	2	fish	3	tanks	4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.0049 VP → V NP 0.105 S → NP VP 0.00126					
1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189			
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → NP VP 0.00378	
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

```
//handle unaries
boolean added = true
while added
  added = false
  for A, B in nonterms
    prob = P(A->B)*score[begin][end][B];
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = B
      added = true
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	0 fish	1 people	2 fish	3 tanks	4
1	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105			
2		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189		
3			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
4				N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	

```
for split = begin+1 to end-1
   for A,B,C in nonterms
      prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
      if prob > score[begin][end][A]
         score[begin]end][A] = prob
         back[begin][end][A] = new Triple(split,B,C)
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish	1	people	2	fish	3	tanks	4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105		NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882			
1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189			
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

```
for split = begin+1 to end-1
   for A,B,C in nonterms
      prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
      if prob > score[begin][end][A]
         score[begin]end][A] = prob
         back[begin][end][A] = new Triple(split,B,C)
```

S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0

N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish	1	people	2	fish	3	tanks	4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105		NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882			
1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189		NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323	
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

```
for split = begin+1 to end-1
   for A,B,C in nonterms
      prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
      if prob > score[begin][end][A]
         score[begin]end][A] = prob
         back[begin][end][A] = new Triple(split,B,C)
```

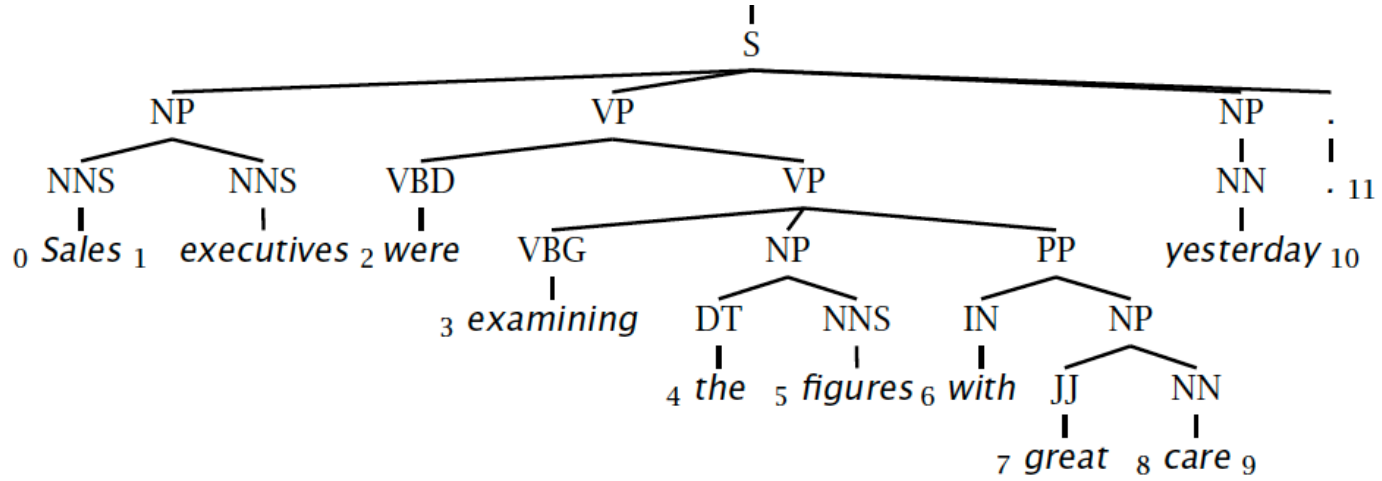
S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0
N → *people*	0.5
N → *fish*	0.2
N → *tanks*	0.2
N → *rods*	0.1
V → *people*	0.1
V → *fish*	0.6
V → *tanks*	0.3
P → *with*	1.0

	fish	1	people	2	fish	3	tanks	4
0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105		NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882		NP → NP NP 0.0000009604 VP → V NP 0.00002058 S → NP VP 0.00018522	
1			N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001		NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189		NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323	
2					N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006		NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
3							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	
4								

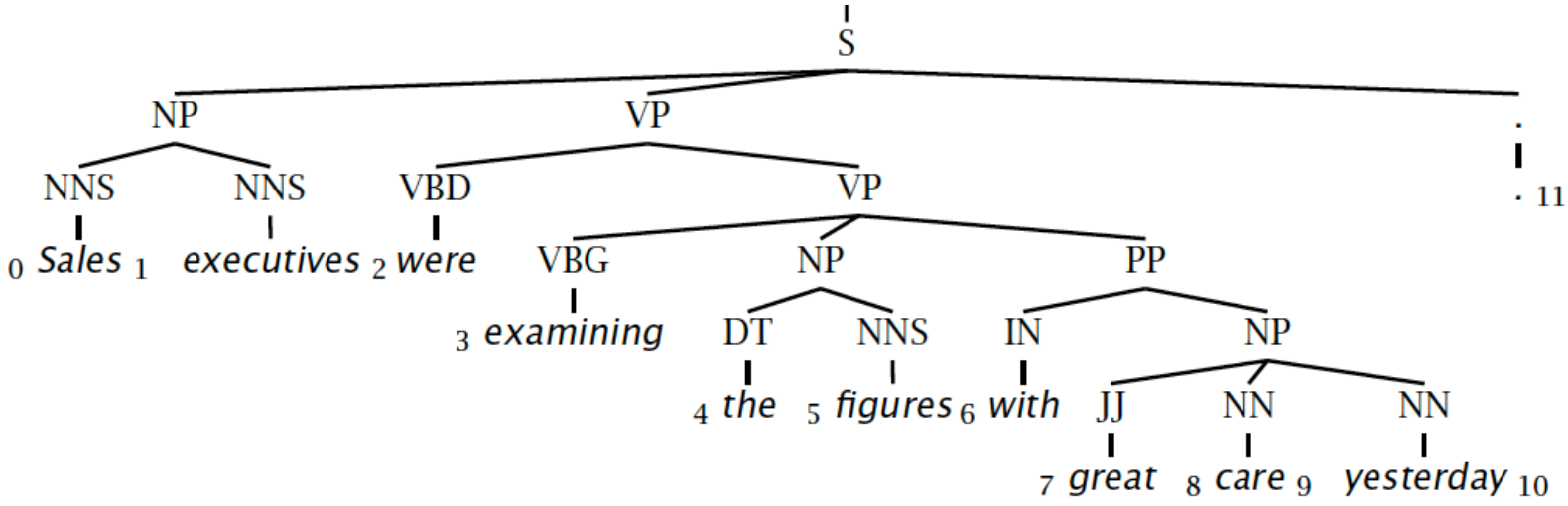
Call buildTree(score, back) to get the best parse

Evaluating constituency parsing

Gold standard brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)



Evaluating constituency parsing

Gold standard brackets:

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)

Candidate brackets:

S-(0:11), **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)

Labeled Precision	3/7 = 42.9%
Labeled Recall	3/8 = 37.5%
LP/LR F1	40.0%
Tagging Accuracy	11/11 = 100.0%