

Automata Theory and Computability

Assignment 1

(Due on Tue 08 Sep 2015)

1. Give DFA's for each of the following languages over $\Sigma = \{a, b\}$:
 - (a) All strings which do *not* contain the substring *babb*. (By a *substring* we mean a contiguous sequence of letters: thus *u* is a *substring* of *w* if there exist strings v_1 and v_2 such that $w = v_1uv_2$.)
 - (b) Between every pair of *b*'s there is at least one *a*.
 - (c) Strings which contain both *aa* and *bb* as substrings.

2. For a language L define

$$\text{first-halves}(L) = \{x \mid \exists y : |x| = |y| \text{ and } xy \in L\}$$

Prove or disprove: if L is regular, then $\text{first-halves}(L)$ is regular.

3. For a set of natural numbers A , define $\text{binary}(A)$ to be the set of binary representations of numbers in A . Similarly define $\text{unary}(A)$ to be the set of "unary" representations of numbers in A : $\text{unary}(A) = \{1^n \mid n \in A\}$. Thus for $A = \{2, 3, 6\}$, $\text{binary}(A) = \{10, 11, 110\}$ and $\text{unary}(A) = \{11, 111, 111111\}$.

Consider the two propositions below:

- (a) For all A , if $\text{binary}(A)$ is regular then so is $\text{unary}(A)$.
- (b) For all A , if $\text{unary}(A)$ is regular then so is $\text{binary}(A)$.

One of the statements above is true and the other is false. Which is true and which is false? Justify your answer.

4. Describe the language defined by the MSO sentence below, as a regular expression. Assume the alphabet to be $\{a, b\}$.

$$\forall x(Q_b(x) \implies \exists y(y < x \wedge Q_a(y))).$$

5. Give an MSO sentence describing the language $(a^+b)^*$.
6. Construct an automaton that accepts all the satisfying assignment of the Presburger formula $x > 2y + 1$, using the procedure described in class.
7. Show the steps in the inductive construction described in class to construct an automaton for the Presburger logic formula

$$\forall x \exists y (x = 2y \vee x = 2y + 1).$$

Draw the formula tree and show the automata associated with each node in the tree.

Examine the final automaton and tell if the given formula is valid or not.