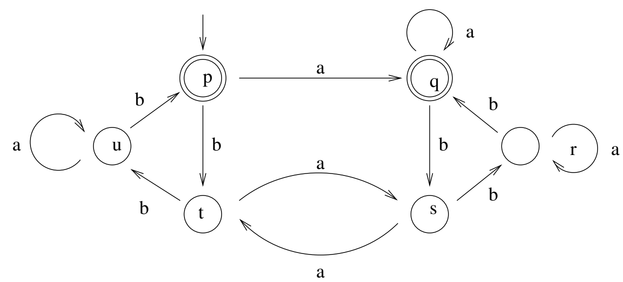
Automata Theory and Computability

Assignment 2

(Due on Mon 5 October 2015)

1. Consider the language $(ab)^*$.
    (a) Describe the equivalence classes of the canonical Myhill-Nerode relation for this language.
    (b) Describe the equivalence classes of the syntactic congruence for this language.
    (c) Describe the syntactic monoid $M(L)$ for this language.
2. Minimize the following DFA using the algorithm described in class:



3. Give a language L over the alphabet $\{a, b\}$ such that L is the union of infinitely many equivalence classes of its canonical Myhill-Nerode relation $\equiv_L$. Describe the equivalence classes.
4. Show that the definition of recognition by monoids is closed under concatenation. More precisely, show how, given monoids M_1 and M_2 that accept languages L_1 and L_2 via morphisms and state-set pairs (φ_1, X_1) and (φ_2, X_2) respectively, to directly construct a monoid recognizing $L_1 \cdot L_2$.
5. Give $FO(<)$-sentences, counter-free DFA's, and star-free regular expressions for the languages below:
    (a) $\{\epsilon\}$
    (b) Strings over $\{a, b\}$ which contain the factor ab but not the factor ba.
    (c) $(ab + ba)^*$.
6. Prove that a regular language L has a counter-free DFA accepting it, iff the canonical automaton $\mathcal{A}_{\equiv_L}$ for L is counter-free.
7. Give a procedure to check whether a given DFA is counter-free or not.

8. Prove that every star-free language is $FO(<)$-definable.

   Hint: Show inductively that for every star-free expression s we can associate an $FO(<)$-formula $\varphi_s(x, y)$, with two free variables x, y, such that for any word w, and positions i and j of w, with $0 \leq i \leq j \leq |w|$, we have:

$$w, [x \mapsto i, y \mapsto j] \models \varphi_s(x, y)$$

   iff the subword $w[i, j] \in L(s)$.