Automata Theory and Computability

Assignment 3 (Context-Free Grammars, PDA's)

(Due on Mon 9 Nov 2015)

1. Consider the language BP_2 of "balanced parenthesis" over the alphabet $\{(,),[,]\}$. For example, the string "(( )[ ])" is in the language but not "([])". Thus BP_2 is similar to BP except that the type of a closing bracket must match the type of the last unmatched opening bracket. Give a CFG for BP_2. There is no need to prove your answer correct, but check your answer by doing an informal proof of correctness.

2. Consider the CFG G below:

$$\begin{array}{lcl} S & \to & aSb \mid aA \mid Bb \\ A & \to & aA \mid \epsilon \\ B & \to & Bb \mid \epsilon. \end{array}$$

   (a) Describe the language accepted by G.

   (b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is, first identify the basic pumps for G, and the $\leq$-minimal parse trees. Use these to obtain an expression for $\psi(L(G))$.

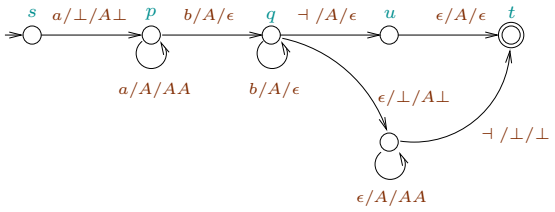
   (c) Use the semi-linear expression above to give a regular expression that is letter-equivalent to $L(G)$.

3. Give the state diagram of PDA's for the following languages:

   (a) $\{w \in \{a,b\}^* \mid \#_a(w) = 2 \cdot \#_b(w)\}$.

   (b) The *complement* of the language $\{ww \mid w \in \{a,b\}^*\}$.

4. Consider the DPDA $\mathcal{M}$ given below.



   Follow the steps discussed in class to construct a DPDA $\mathcal{N}$ that accepts the complement of the language accepted by $\mathcal{M}$.

   (a) First construct a language-equivalent PDA $\mathcal{M}'$ that has sink accepting states, and a single reject state r'. $\mathcal{M}'$ must read all its inputs, except possibly for inputs that cause it to enter into an infinite sequence of ϵ-moves.

(b) Use the pushdown reachability algorithm to find the spurious transitions in $\mathcal{M}'$. Describe the PDS and set of configurations C you consider. Apply the algorithm for $Pre^*(C)$, showing the saturated P-automaton.

(c) Identify the spurious transitions. Describe the DPDA $\mathcal{M}''$ you obtain by "removing" the spurious transitions.

(d) Describe the complemented DPDA $\mathcal{N}$ obtained from $\mathcal{M}''$.

5. Let a and b be symbols in an alphabet A. A language L over A satisfies property $P_{a,b}$ iff: whenever there exists strings u and v such that $uav \in L$, there also exists a string v' such that $ubv' \in L$.

   Show that given a deterministic PDA $\mathcal{M}$ over an alphabet A, and symbols a and b in A, we can check whether $L(\mathcal{M})$ satisfies property $P_{a,b}$.