

# Visibly Pushdown Automata

Deepak D'Souza

Department of Computer Science and Automation  
Indian Institute of Science, Bangalore.

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# Outline

- 1 Visibly Pushdown Automata
- 2 Closure properties of VPL
- 3 Determinization

# Visibly Pushdown Automata

- A sub-class of Pushdown Automata (PDA's) in which pushing/popping from the stack is dictated by input letters.
- Useful properties for verification
  - Closed under operations like union, intersection, complementation, concatenation, Kleene- $*$ .
  - Decidable language inclusion and universality problems.

# Example VPA

Example VPA for  
 $\{a^n b^n \mid n \geq 0\}$

$$\Sigma_c = \{a\}$$

$$\Sigma_r = \{b\}$$

$$\Sigma_{int} = \emptyset$$

$(s, a, p, \perp)$

$(p, a, p, A)$

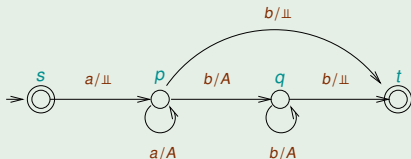
$(p, b, A, q)$

$(q, b, A, q)$

$(q, b, \perp, t)$

$F = \{s, t\}$ .

State Diagram of VPA



# Definitions

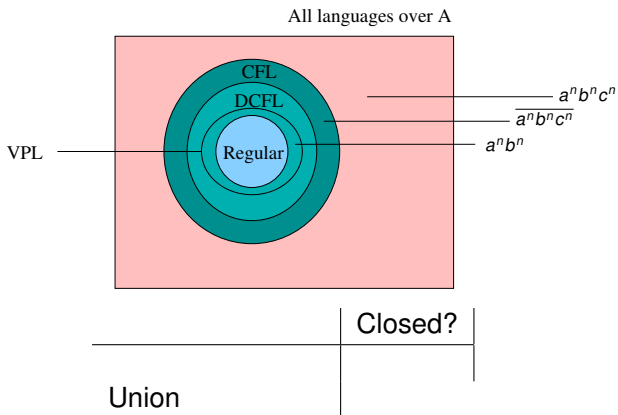
A VPA over a partitioned alphabet  $\tilde{\Sigma} = (\Sigma_c, \Sigma_r, \Sigma_{int})$  is a structure  $M = (Q, Q_0, \Gamma, \perp, \delta, F)$  where  $Q$  is a finite set of states,  $Q_0$  is a set of initial states,  $\Gamma$  is a stack alphabet with  $\perp \in \Gamma$ ,  $F$  is a set of final states, and  $\delta$  is the transition relation of the form:

- $(p, a, q, A)$  if  $a \in \Sigma_c$  (push transition)
- $(p, a, A, q)$  if  $a \in \Sigma_r$  (pop transition)
- $(p, a, q)$  if  $a \in \Sigma_{int}$  (internal transition).

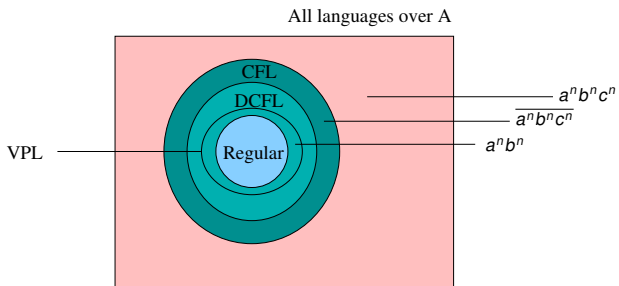
Run of  $M$  on a word  $w = a_1 a_2 \dots a_n$ .

Class of languages accepted by VPA's are called **Visibly Pushdown Languages (VPL)**.

# Closure Properties of VPL

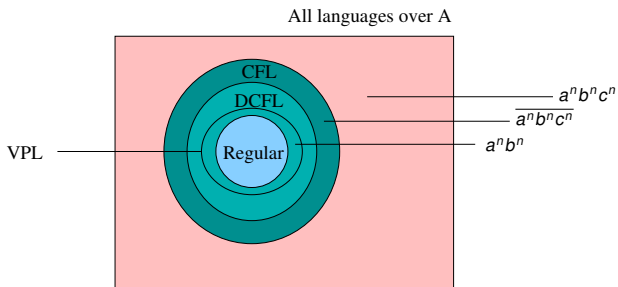


# Closure Properties of VPL



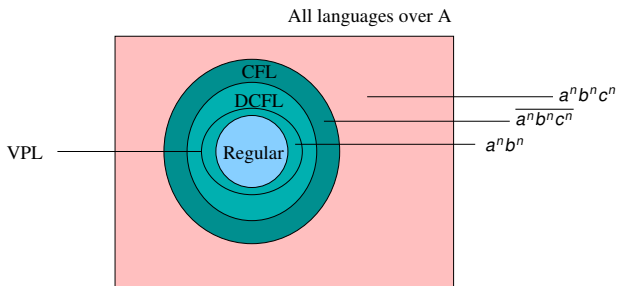
	Closed?
Union	✓
Intersection	

# Closure Properties of VPL



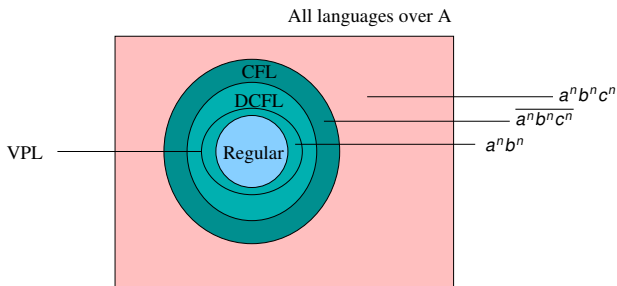
	Closed?
Union	✓
Intersection	✓
Concatenation	

# Closure Properties of VPL



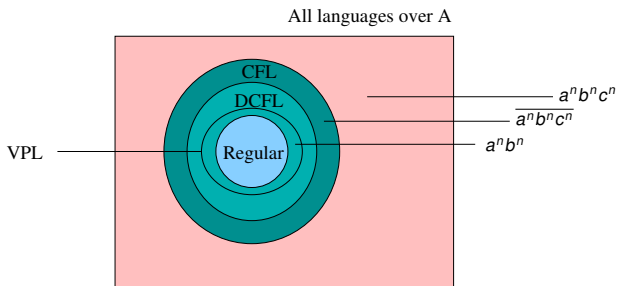
	Closed?
Union	✓
Intersection	✓
Concatenation	✓
Kleene-*	

# Closure Properties of VPL



	Closed?
Union	✓
Intersection	✓
Concatenation	✓
Kleene-*	✓
Complementation	

# Closure Properties of VPL



	Closed?
Union	✓
Intersection	✓
Concatenation	✓
Kleene-*	✓
Complementation	✓

# Determinizing VPA's

Let  $M = (Q, Q_0, \Gamma, \delta, F)$  be a VPA over  $\tilde{\Sigma}$ . We define a new VPA  $M'$  as follows:

- Control state is of the form  $(S, R)$  where  $S \subseteq Q \times Q$  and  $R \subseteq Q$ .
- Stack symbols will be of the form  $(S, R, a)$  where  $S$  and  $R$  are as above, and  $a \in \Sigma_c$  is a call alphabet.

