Department of Computer Science & Automation

Indian Institute of Science

Nested Word Automata

Seminar Presentation
08 Dec 16

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Overview

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Regular language

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procedure foo()
{
    return;
}
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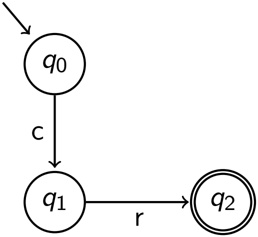

(det.) Context-free language

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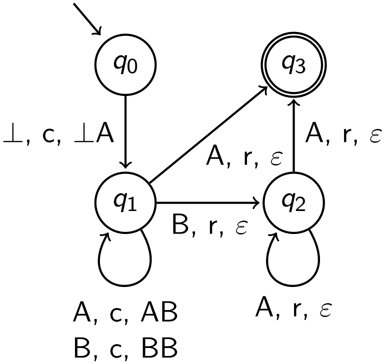

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☺	**regular**	**context-free**
comparison of numbers	constants	two variables
closure		
decidability		
determinize		

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Question: Is there some class of languages in between that is more expressive than regular languages, but keeps their nice properties?

Answer (Alur & Madhusudan 2004): yes, at least in some sense

Visibly pushdown languages (VPLs)

A *visibly pushdown language* (VPL) is the language accepted by a *visibly pushdown automaton* (VPA).

A VPA $\mathcal{A} = \langle Q, q_0, Q_f, \Sigma, \Gamma, \bot, \delta \rangle$ is a deterministic PDA with special rules: Determined by the input symbol, only one symbol per **push** is allowed and reading the stack implies a **pop**.

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Note: pops occur implicitly, $\perp$ never popped, no ε

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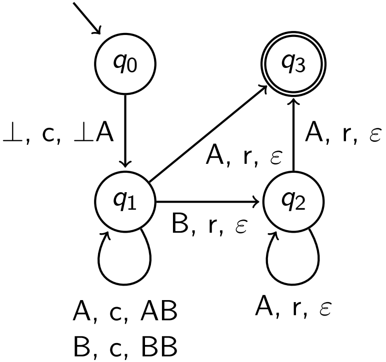

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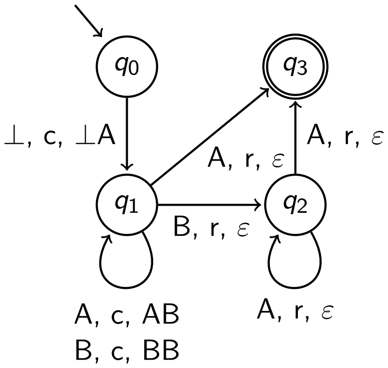

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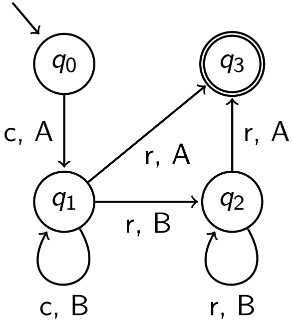

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From VPAs to NWAs

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- automaton model: *nested word automata* (NWAs)
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  $\rightarrow$ NWAs $\preceq$ deterministic PDAs
- main idea: call and return symbols are matched in the input

Overview

Well nested sequences

A sequence of symbols is *well nested* if calls and returns are matched without crossing, i.e., for any different call-return-pairs (c_i, r_i), (c_j, r_j), $c_i < c_j < r_i < r_j$ is forbidden.

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Note: Every sequence has a unique well nesting.

Nested words

A relation $\rightsquigarrow \subset \{-\infty, 1, 2, \ldots, \ell\} \times \{1, 2, \ldots, \ell, \infty\}$ of length $\ell \geq 0$ is a *matching relation* if the following holds:

- I if $i \rightsquigarrow j$, then $i < j$ (monotone)
- II if $i_1 \rightsquigarrow j$ and $i_2 \rightsquigarrow j$, then $i_1 = i_2$ (left-unique)
  if $i \rightsquigarrow j_1$ and $i \rightsquigarrow j_1$, then $j_1 = j_2$ (right-unique)
- III if $i_1 \rightsquigarrow j_1$ and $i_2 \rightsquigarrow j_2$, then we have not $i_1 < i_2 < j_1 < j_2$ (well nested)

Explanation:

- I not **r c**, not reflexive
- II not **c c r**, not **c r r**
- III not **c c r r**

> ex post note: $(-\infty, \infty) \notin \rightsquigarrow$, $\pm\infty$ excluded from uniqueness

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If $i \leadsto j$, i is a *call position* and j is a *return position*. All the rest is an *internal position*. If $i \neq -\infty$ and $j \neq \infty$, they are *well-matched*, otherwise *pending*. $e \in \leadsto$ is a *nesting edge*.

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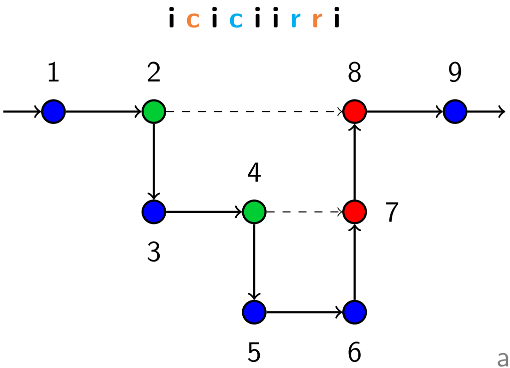
A *nested word* n over Σ is a pair $(a_1 \cdots a_\ell, \rightsquigarrow)$, where $a_i \in \Sigma$ and $\rightsquigarrow$ is a matching relation of length ℓ.

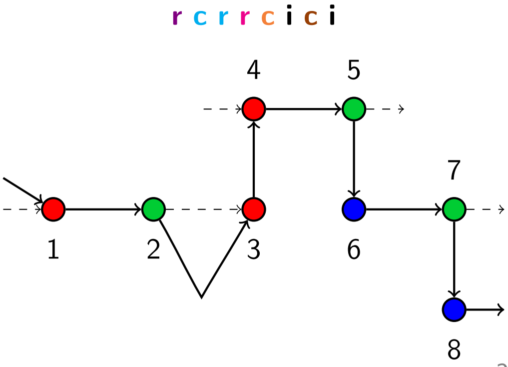
Example 1



adapted from [1]

Here: $2 \rightsquigarrow 8$, $4 \rightsquigarrow 7$ and the whole word is well-matched.

Example 2



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Here: $-\infty \rightsquigarrow 1$, $2 \rightsquigarrow 3$, $-\infty \rightsquigarrow 4, 5 \rightsquigarrow \infty, 7 \rightsquigarrow \infty$ and only $2 \rightsquigarrow 3$ is well-matched.

Definition of NWAs

$\mathcal{A} = \langle Q, q_0, Q_f, P, p_0, P_f, \delta_i, \delta_c, \delta_r \rangle$ over alphabet Σ

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- $\delta_c \subseteq Q \times \Sigma \rightarrow Q \times P$ call transition function,
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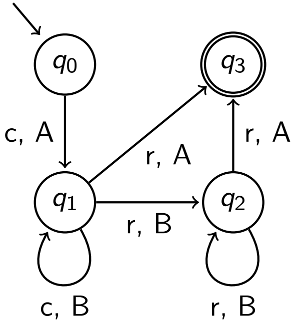
acceptance via both Q_f and P_f
as VPAs: at return implicitly go to hierarchical state before matching call

$\mathcal{L}_2$ as NWA

Consider again $\mathcal{L}_2 = \{c^n\ r^n \mid n > 0\}$.

We construct an NWA for $\mathcal{L}_2' := \{(\langle c)^n\ (r\rangle)^n \mid n > 0\}$.



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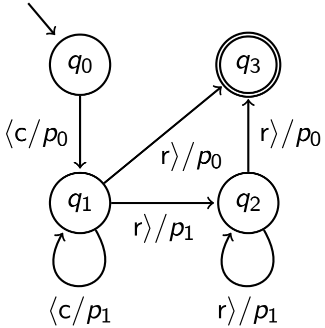

$P = \{p_0, p_1\}$, $P_f \subseteq \{p_0\}$

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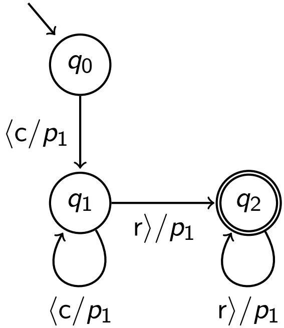
We can also use hierarchical states for acceptance.



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  possibly exponentially more states for deterministic NWAs
- not all sets of NWs acceptable by NWAs
  $\{(\langle a)^n (b\rangle)^n \mid n > 0\}$ vs. $\{a^n b^n \mid n > 0\}$

Comparison of properties

	DFA	DNWA	PDA	DPDA
pre-/suffix	✓	✓	✓	✓
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emptiness	NLOGSPACE	PTIME	PTIME	PTIME
equivalence	NLOGSPACE	PTIME	undecidable	decidable
inclusion	NLOGSPACE	PTIME	undecidable	undecidable

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inclusion	NLOGSPACE	PTIME	undecidable	undecidable

Note: Equivalence and inclusion problem are EXPTIME-complete for nondeterministic NWAs.

Implication: determinization $\in \Omega(\text{EXPTIME})$ if at all possible

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  $\rightarrow$ powerset construction over nesting edges
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- NWAs: also need information about hierarchical states
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  hierarchical states $=$ nesting edges $+$ call symbol so far
- handle hierarchical proceeding when reading return symbols

The states: definition

Consider the NNWA $\mathcal{A} = \langle Q, Q_0, Q_f, P, P_0, \overbrace{P_f}^{\overset{\text{wlog}}{=P}}, \delta_i, \delta_c, \delta_r \rangle$.
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Consider a nested word n with k pending calls. We can write this

$$n = n_1 \langle c_1 n_2 \langle c_2 \cdots n_k \langle c_k n_{k+1}$$

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i.e., $\exists q, q'.(q, q') \in S_{k+1} \wedge q \xrightarrow{n_{k+1}}_{\mathcal{A}} q' \wedge q' \in Q_f$

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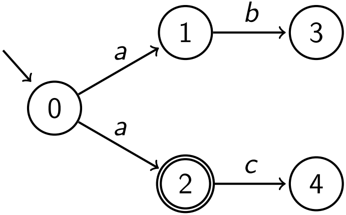
$$\delta_i'(S_{k+1}, i) = \{(q, q'') \mid (q, q') \in S_{k+1} \wedge q'' \in \delta_i(q', i)\}$$

Example



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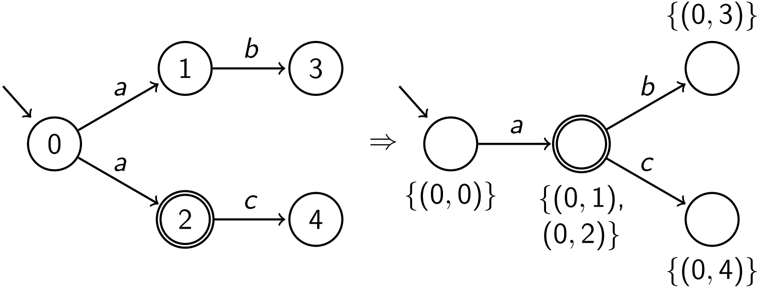

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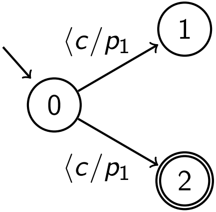

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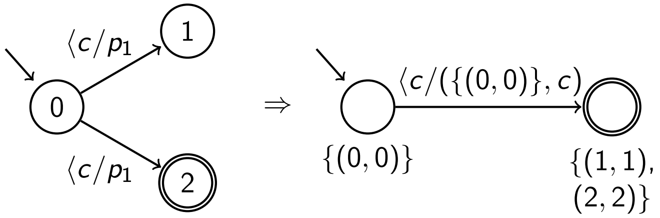

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We have two cases here:

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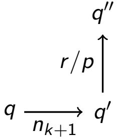

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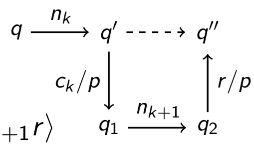

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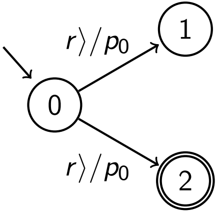

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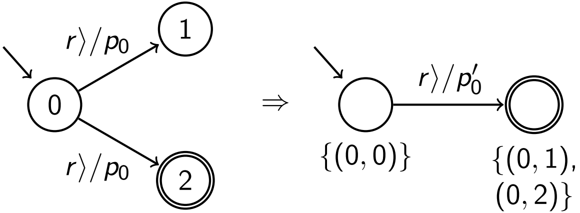

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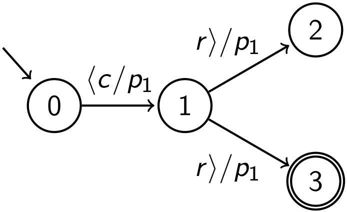

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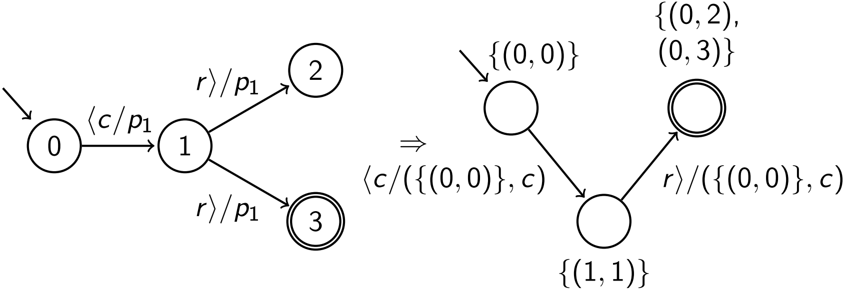
0
$\langle c/p_1$
1
$r\rangle/p_1$
2
$r\rangle/p_1$
3
$\Rightarrow$
$\{(0,0)\}$
$\langle c/(\{(0,0)\},c)$
$\{(1,1)\}$
$r\rangle/(\{(0,0)\},c)$
$\{(0,2),$
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  This is succinct, so there exists an example where the DNWA cannot have less states.

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- recent concept, time will show the relevance

References

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