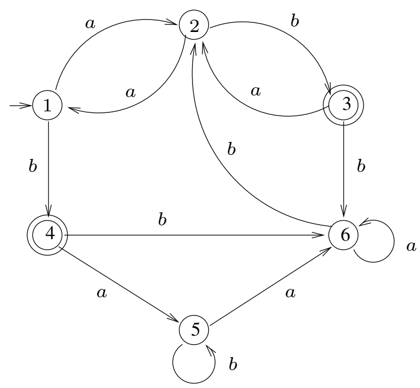
**Automata Theory and Computability**

*Assignment 2*

(Due on Fri 30 September 2016)

1. Minimize the following DFA using the algorithm described in class:




2. Describe the equivalence classes of the Myhill-Nerode relation $\equiv_L$ for the language $L = \{x \in \{a,b\}^* \mid \#_a(x) = \#_b(x)\}$. Use your answer to prove that $L$ is not regular.

3. Consider the language $(a+b)^*ab(a+b)^*$.

   (a) Describe the equivalence classes of the canonical Myhill-Nerode relation for this language.

   (b) Describe the syntactic monoid $M(L)$ for this language.

   (c) Describe the equivalence classes of the syntactic congruence for this language.

4. We know that the syntactic congruence $\cong_L$ of a language $L$ refines the canonical MN relation $\equiv_L$ of $L$. If $\equiv_L$ has $n$ classes, what is the maximum number of classes that $\cong_L$ can have? Give a language $L$ that achieves this number.

5. Show that the class of languages over an alphabet $A$ that are recognizable by finite monoids, are closed under the prefix operation. Recall that for a language $L \subseteq A^*$, the language of prefixes of $L$, denoted *pref*$(L)$, is defined to be the set $\{u \in A^* \mid \exists v \in A^* : \ u \cdot v \in L\}$.

6. Show that the class of languages over an alphabet $A$ that are recognizable by finite monoids, are closed under concatenation. More precisely, show how, given finite monoids $M_1$ and $M_2$ that accept languages $L_1$ and $L_2$ via morphisms and state-set pairs $(\varphi_1, X_1)$ and $(\varphi_2, X_2)$ respectively, we can directly construct a monoid recognizing $L_1 \cdot L_2$.

7. Give $FO(<)$-sentences, counter-free DFA's, and star-free regular expressions for the languages below:
   (a) $\{\epsilon\}$
   (b) $a^*b^*$
   (c) Strings over $\{a, b\}$ which contain the factor $ab$ but not the factor $ba$.
8. Give a procedure to check whether a given DFA is counter-free or not.