Automata Theory and Computability

Assignment 3 (Context-Free Grammars, PDAs)

(Due on Fri 18 Nov 2016)

1. Consider the language BP_2 of "balanced parenthesis" over the alphabet $\{(,),[,]\}$. For example, the string "(()[])" is in the language but not "([)]". Thus BP_2 is similar to BP except that the type of a closing bracket must match the type of the last unmatched opening bracket. Give a CFG for BP_2. There is no need to prove your answer correct, but check your answer by doing an informal proof of correctness.

2. Consider the CFG G below:

$$\begin{array}{rcl} S & \rightarrow & aSb \mid aA \mid Bb \\ A & \rightarrow & aA \mid \epsilon \\ B & \rightarrow & Bb \mid \epsilon. \end{array}$$

   (a) Describe the language accepted by G.

   (b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is, first identify the basic pumps for G, and the $\leq$-minimal parse trees. Use these to obtain an expression for $\psi(L(G))$.

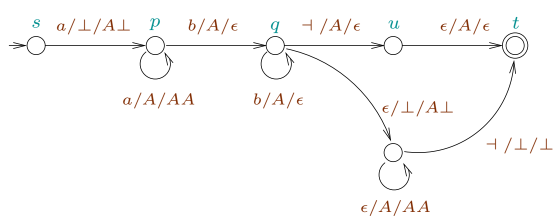
   (c) Use the semi-linear expression above to give a regular expression that is letter-equivalent to $L(G)$.

3. Give the state diagram of PDAs for the following languages:

   (a) $\{w \in \{a,b\}^* \mid \#_a(w) \geq 2 \cdot \#_b(w)\}$.

   (b) The *complement* of the language $\{ww \mid w \in \{a,b\}^*\}$.

4. Consider the DPDA $\mathcal{M}$ given below.



Follow the steps discussed in class to construct a DPDA $\mathcal{N}$ that accepts the complement of the language accepted by $\mathcal{M}$.

   (a) First construct a language-equivalent PDA $\mathcal{M}'$ that has sink accepting states, and a single reject state r'. $\mathcal{M}'$ must read all its inputs, except possibly for inputs that cause it to enter into an infinite sequence of ϵ-moves.

(b) Use the pushdown reachability algorithm to find the spurious transitions in $\mathcal{M}'$. Describe the PDS and set of configurations C you consider. Apply the algorithm for $Pre^*(C)$, showing the saturated P-automaton.

(c) Identify the spurious transitions. Describe the DPDA $\mathcal{M}''$ you obtain by "removing" the spurious transitions.

(d) Describe the complemented DPDA $\mathcal{N}$ obtained from $\mathcal{M}''$.

5. Describe how you would check whether a given PDA accepts a non-empty language using the pushdown reachability algorithm.

6. Prove that the class of DCFLs are *not* the largest subset of CFL's that are closed under complementation. (*Hint*: Exhibit a language L such that both L and its complement are accepted by non-deterministic PDAs but not by DPDAs. You can give an informal justification for why they are not accepted by any DPDA).

7. Is the class of DCFLs closed under concatenation? Justify your answer.

8. (a) Prove that PDAs are closed under the prefix operation. That is, given a PDA M you can construct a PDA M' that accepts the prefix closure of $L(M)$.

   (b) Prove similarly that DPDAs are closed under prefixes.