

Automata Theory and Computability

Assignment 4 (PDA's and Decidability)

Due on Fri 2nd December 2016.

1. Show that the following function is computable by a Turing Machine in the sense discussed in class. Give the complete state-transition diagram of the TM.

(integer division) $div : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $div(m, n)$ is the largest integer less than or equal to m/n if $n > 0$, and 0 otherwise.

2. Show that it is undecidable whether a given Turing machine accepts *some* string at all (i.e. whether its language is non-empty or not).
3. Prove that a language L is recursive iff it can be enumerated in increasing lexicographic order.

Using this fact, prove that every infinite r.e. language must have an infinite recursive subset.

4. Show that neither the language

$$\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$$

nor its complement is r.e.

5. One of the following problems is decidable and the other is not. Which is which? Justify your answer.

(a) Given a CFL L and a regular language R , is $L \subseteq R$?

(b) Given a CFL L and a regular language R , is $R \subseteq L$?

6. Show that it is undecidable to check whether the intersection of two given CFLs is a CFL.
7. Is it decidable to check whether the language of a given PDA P is contained in that of a given VPA V ? Justify your answer.