**Automata Theory and Computability**

*Assignment 2*

(Due on Thu 4th Oct 2018)

1. Show that the language $L$ over the alpbabet $\{a, b\}$ comprising strings with an even number of $a$'s and an odd number of $b$'s, is recognizable by a finite monoid.

2. Show that the class of languages over an alphabet $A$ that are recognizable by finite monoids, are closed under intersection. More precisely, show how, given finite monoids $M_1$ and $M_2$ that accept languages $L_1$ and $L_2$ via morphisms and state-set pairs $\varphi_1, X_1$ and $\varphi_2, X_2$ respectively, to directly construct a monoid recognizing $L_1 \cap L_2$.

3. Consider the language $L$ given by the regular expression $(a+b)^*ab(a+b)^*$.

   (a) Describe the equivalence classes of the canonical Myhill-Nerode relation for this language.

   (b) Describe the canonical DFA for this language.

   (c) Describe the syntactic monoid $M(L)$ for this language.

   (d) Show that $L$ is recognizable by a finite monoid.

   (e) Describe the equivalence classes of the syntactic congruence for this language.

4. Give a language $L$ over the alphabet $\{a, b, c\}$ for which the syntactic congruence $\cong_L$ is exponentially more refined than the canonical MN relation $\equiv_L$.

5. For every $k \geq 0$, let $L_k \subseteq \{a, b\}^*$ be the language of strings in which the difference between the number of $a$'s and $b$'s is at most $k$. Define a family of CFGs $G_k$ such that $\forall k, L(G_k) = L_k$. Sketch a proof that argues why your grammars are correct.

6. Prove that the language $\{a^m b^n a^{m+n} \mid m, n \geq 0\}$ is context-free.