

Automata Theory and Computability

Assignment 4 (VPAs and Decidability)

Due on Mon 3rd December 2018.

1. Let $\hat{A} = \langle A_{\text{call}}, A_{\text{ret}}, A_{\text{int}} \rangle$ be a pushdown alphabet. Let $A = A_{\text{call}} \cup A_{\text{ret}} \cup A_{\text{int}}$. We say that string $w \in A^*$ is well-matched if:
 - w has an equal number of letters from A_{call} and A_{ret} , and
 - in every prefix of w , the number of letters from A_{call} is greater than or equal to the number of letters from A_{ret} .

Let $L \subseteq A^*$. Define the syntactic congruence \approx_L on well-matched strings as: $w_1 \approx_L w_2$ iff $\forall x, y \in A^*$, $xw_1y \in L$ iff $xw_2y \in L$.

- (a) If L is a VPL, then it can be shown that \approx_L has finite index. Using this fact, prove that $L = \{a^{2n}b^n \mid n \geq 0\}$ is not a VPL.
 - (b) For $k \geq 0$, define $L_k = \{a^{n+k}b^n \mid n \geq 0\}$. Show that L_k is a VPL by constructing a VPA for L_k . (Just give the general construction in terms of k – no proof is needed.)
2. Show that the following function is computable by a Turing Machine in the sense discussed in class. Give the complete state-transition diagram of the TM.

(integer division) $div : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $div(m, n)$ is the largest integer less than or equal to m/n if $n > 0$, and 0 otherwise.

3. Give the state-transition diagram of a TM that accepts the language $L = \{ww \mid w \in \{a, b\}^*\}$.
4. Is the following problem decidable?

Given a TM M and a string y , does M ever write the symbol $\#$ on its tape on input y ?

Justify your answer as usual.

5. Prove that a language L is recursive iff it can be enumerated in increasing lexicographic order.
Using this fact, prove that every infinite r.e. language must have an infinite recursive subset.

6. Let REG be the language

$$\{M \mid L(M) \text{ is regular} \}.$$

Show without using Rice's theorem that neither REG nor its complement is recursively enumerable.

7. Is it decidable whether a given CFG generates a regular language? Justify your answer.