
Angluin's Algorithm

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The Learner L^*

Initialize S and E to $\{\lambda\}$.

Ask membership queries for λ and each $a \in A$.

Construct the initial observation table (S, E, T)

Repeat:

 While (S, E, T) is not closed or not consistent:

 If (S, E, T) is not consistent,

 then find s_1 and s_2 in S , $a \in A$, and $e \in E$ such that

$\text{row}(s_1) = \text{row}(s_2)$ and $T(s_1.a.e) \neq T(s_2.a.e)$,

 add $a.e$ to E ,

 and extend T to $(S \cup S.A).E$ using membership queries.

 If (S, E, T) is not closed.

 then find $s_1 \in S$ and $a \in A$ such that

$\text{row}(s_1.a)$ is different from $\text{row}(s)$ for all $s \in S$,

 add $s_1.a$ to S ,

 and extend T to $(S \cup S.A).E$ using membership queries.

Once (S, E, T) is closed and consistent. let $M = M(S, E, T)$.

Make the conjecture M .

If the Teacher replies with a counter-example t , then

 add t and all its prefixes to S

 and extend T to $(S \cup S.A).E$ using membership queries.

Until the Teacher replies yes to the conjecture M .

Halt and output M .

Lets run the algorithm, taking the unknown regular set to be

$$U = \{ w \mid \#_a(w) \bmod 3 = 0 \}$$

I

Initialize S and E to $\{\lambda\}$.

Ask membership queries for λ and each $a \in A$.

Construct the initial observation table (S, E, T)

$$S = \{\lambda\}$$

$$E = \{\lambda\}$$

I

Initialize S and E to $\{\lambda\}$.

Ask membership queries for λ and each $a \in A$.

Construct the initial observation table (S, E, T)

$$T(\lambda) = ?$$

$$T(a) = ?$$

$$T(b) = ?$$

I

Initialize S and E to $\{\lambda\}$.

Ask membership queries for λ and each $a \in A$.

Construct the initial observation table (S, E, T)

	λ
λ	1
a	0
b	1

II

Repeat:

While (S, E, T) is not closed or not consistent:

If (S, E, T) is not consistent,
then find s_1 and s_2 in S , $a \in A$, and $e \in E$ such that $\text{row}(s_1) = \text{row}(s_2)$ and $T(s_1.a.e) \neq T(s_2.a.e)$,
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add $s_1.a$ to S ,
and extend T to $(S \cup S.A).E$ using membership queries.

	λ
λ	1
<u>a</u>	0
b	1

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Repeat:

While (S, E, T) is not closed or not consistent:

If (S, E, T) is not consistent,
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and extend T to $(S \cup S.A).E$ using membership queries.

	λ
λ	1
a	0
b	1
aa	0
ab	0

III

Make the conjecture M.

If the Teacher replies with a counter-example t , then

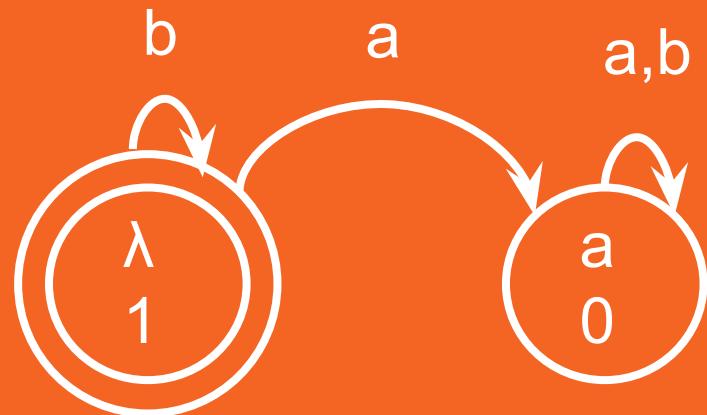
add t and all its prefixes to S

and extend T to $(S \cup S.A).E$ using membership queries.

Repeat until the Teacher replies yes to the conjecture M.

	λ
λ	1
a	0
b	1
aa	0
ab	0

$Q = \{\text{row}(s) : s \in S\},$
 $q_0 = \text{row}(\lambda),$
 $F = \{ \text{row}(s) : s \in S \text{ and } T(s) = I \},$
 $\delta(\text{row}(s), a) = \text{row}(s.a)$



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Make the conjecture M.

If the Teacher replies with a counter-example t, then

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$t = aaa$

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Repeat until the Teacher replies yes to the conjecture M.

	λ
λ	1
a	0
aa	0
aaa	
b	1
ab	0
aab	
aaaa	
aaab	

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Make the conjecture M.

If the Teacher replies with a counter-example t , then

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	λ
λ	1
a	0
aa	0
aaa	1
b	1
ab	0
aab	0
aaaa	0
aaab	1

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	λ
λ	1
a	0
aa	0
aaa	1
b	1
ab	0
aab	0
aaaa	0
aaab	1

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Repeat:

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	λ	a
λ	1	
a	0	
aa	0	
aaa	1	
b	1	
ab	0	
aab	0	
$aaaa$	0	
$aaab$	1	

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	λ	a
λ	1	0
a	0	0
aa	0	1
aaa	1	0
b	1	0
ab	0	0
aab	0	1
$aaaa$	0	0
$aaab$	1	0

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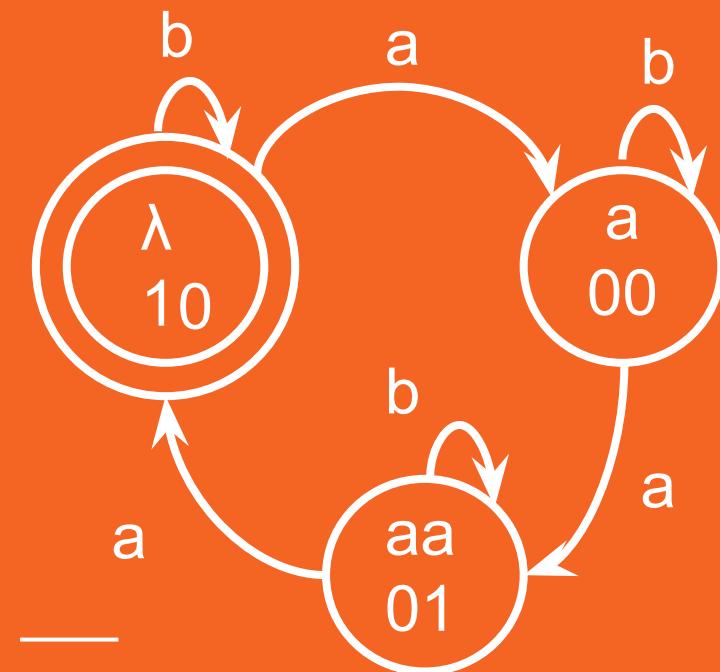
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	λ	a
λ	1	0
a	0	0
aa	0	1
aaa	1	0
b	1	0
ab	0	0
aab	0	1
aaaa	0	0
aaab	1	0

	λ	a
λ	1	0
a	0	0
aa	0	1
aaa	1	0
b	0	0
ab	0	0
aab	0	1
aaaa	0	0
aaab	1	0

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