Automata Theory and Computability

Assignment 2

(Due on Fri 27th September 2019)

1. Describe the classes of the canonical Myhill-Nerode relation for the language L over $\{a, b\}$, comprising strings with an odd number of a's and at most one b. Use it to construct the canonical DFA for L.

2. Describe the equivalence classes of the canonical Myhill-Nerode relation $\equiv_L$ for the language $L = \{x \in \{a,b\}^* \mid \#_a(x) = \#_b(x)\}$. Use this to argue that L is not regular. 2016

3. Consider the monoid $M = (\{1, m, m'\}, \circ, 1)$ where $\circ$ is given by:

$\circ$	1	m	m'
1	1	m	m'
m	m	1	m'
m'	m'	m'	m'

   (a) Let A be the alphabet $\{a, b\}$. Describe a morphism $\varphi : A^* \to M$, and a subset X of M, such that the language recognized by M, φ, and X, is the language $\{a^k \mid k \text{ is odd}\}$.

   (b) Describe the syntactic monoid of this language.

4. Describe the syntactic monoid of the singleton language $L = \{ab\}$ over the alphabet $\{a, b\}$.

5. Suppose a language $L \subseteq A^*$ is recognized by a finite monoid $(M, \circ)$ via a morphism, final-set pair (φ, X). Show how to construct a DFA $\mathcal{A}$ that accepts L.

6. Give any language (other than the one given in class) whose synctactic congruence strictly refines its canonical MN relation.

7. Show that the class of languages over an alphabet A that are recognizable by finite monoids, are closed under union. More precisely, show how, given finite monoids M_1 and M_2 that accept languages L_1 and L_2 via morphisms and state-set pairs φ_1, X_1 and φ_2, X_2 respectively, to directly construct a monoid recognizing $L_1 \cup L_2$.

8. In a similar way to the question above, show that the class of languages recognizable by finite monoids is closed under the suffix operation. In other words, for a language L over an alphabet A, define

$$\mathit{suffix}(L) = \{v \in A^* \mid \exists u \in A^* : \; u \cdot v \in L\}.$$

   Now show that if L is recognizable by a finite monoid, so is $\mathit{suffix}(L)$.