Automata Theory and Computability

Assignment 3 (Ultimate Periodicity, CFGs, Parikh's Theorem, PDAs)

(Due on Mon 4 Nov 2019)

1. Using results about regular languages and ultimate periodicity, or otherwise, show that if $L \subseteq \{a\}^*$, then L^* must be regular.

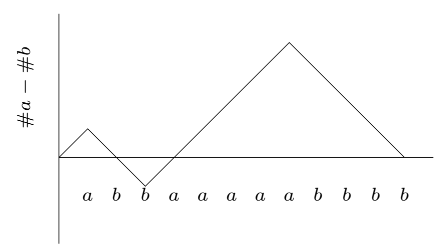
2. Give a language L over the alphabet $\{a, b\}$ which satisfies the property that neither L nor its complement contains an infinite regular language.

3. Consider the grammar G below:

$$S \longrightarrow SS \mid aSb \mid bSa \mid \epsilon.$$

Prove that it generates the language $\{x \in \{a,b\}^* \mid \#_a(x) = \#_b(x)\}$.

Hint: Consider the graph of a word x where you plot the value $\#_a(y) - \#_b(y)$ against prefixes y of x. Use induction as usual.



4. Consider the language $L = \{a^n b^{n^2} \mid n \geq 0\}$. Show that L is not a CFL using:

   (a) the Pumping Lemma for CFLs.

   (b) Parikh's Theorem.

5. Consider the CFG G below:

$$\begin{array}{lcl} S & \rightarrow & XC \mid AY \\ X & \rightarrow & aXb \mid ab \\ Y & \rightarrow & bYc \mid bc \\ A & \rightarrow & aA \mid a \\ C & \rightarrow & cC \mid c \end{array}$$

   (a) Describe the language accepted by G.

   (b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is

      i. Identify the basic pumps for G.

      ii. Identify the $\leq$-minimal parse trees.

iii. Use these to obtain an expression for $\psi(L(G))$.

(c) Use the semi-linear expression obtained above to give a regular expression that is letter-equivalent to $L(G)$.

6. Construct a PDA that accepts the *complement* of the language

$$\{ww \mid w \in \{a,b\}^*\}$$

using the idea discussed in class. Give your PDA in a diagrammatic form.

7. Consider the PDA below that accepts by final state, where the set of final states is $\{t\}$.

$$\begin{array}{lcl}
(s,a,\bot) & \rightarrow & (p,A\bot) \\
(p,a,A) & \rightarrow & (p,AA) \\
(p,b,A) & \rightarrow & (q,\epsilon). \\
(p,c,A) & \rightarrow & (r,\epsilon). \\
(q,b,A) & \rightarrow & (q,\epsilon). \\
(r,c,A) & \rightarrow & (r,\epsilon). \\
(q,b,\bot) & \rightarrow & (t,\epsilon). \\
(r,c,\bot) & \rightarrow & (t,\epsilon). \\
(t,-,-) & \rightarrow & (t,\epsilon).
\end{array}$$

(a) Describe the language accepted by the PDA.

(b) Use the construction described in class to construct a single-state PDA that accepts the same language.