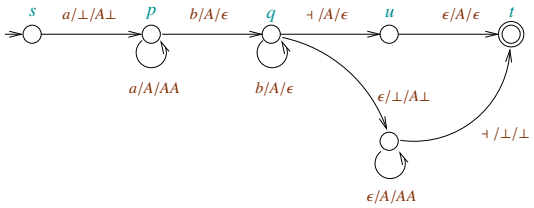
Automata Theory and Computability

Assignment 4 (Pushdown Reachability, DPDAs, VPAs, Turing Machines)

(Total Marks 55. Due on Mon 18 Nov 2019)

1. Consider the DPDA M given below. (*15*)



   Follow the steps discussed in class to construct a DPDA N that accepts the complement of the language accepted by M.

   (a) First construct a language-equivalent PDA M' that has sink accepting states, and a single reject state r'. M' must read all its inputs, except possibly for inputs that cause it to enter into an infinite sequence of ϵ-moves.

   (b) Use the pushdown reachability algorithm to find the spurious transitions in M'. Describe the PDS and set of configurations C you consider. Apply the algorithm for $Pre^*(C)$, showing the saturated P-automaton.

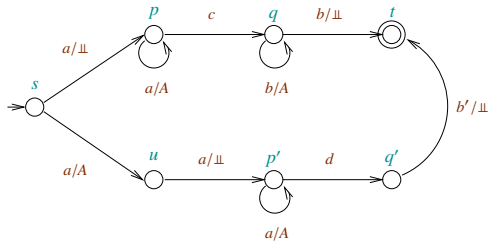
   (c) Identify the spurious transitions. Describe the DPDA M'' you obtain by "removing" the spurious transitions.

   (d) Describe the complemented DPDA N obtained from M''.

2. Describe a procedure similar to the one for pre^*, for computing $post^*$ for a given regular set C of configurations of a pushdown system P. (*10*)

3. Let $\widetilde{\Sigma} = (\Sigma_c, \Sigma_r, \Sigma_i)$ be a partitioned alphabet, with $\Sigma = \Sigma_c, \Sigma_r, \Sigma_i$. Show that if $L \subseteq \Sigma^*$ is a Visibly Pushdown Language (VPL) over $\widetilde{\Sigma}$ then so is L^*. (*10*)

4. Consider the VPA below over the partitioned alphabet $(\{a\}, \{b, b'\}, \{c, d\})$. (*10*)



   (a) Describe the language accepted by the VPA.

(b) Use the construction discussed in class to determinize it.

5. Show that the following functions are computable by a Turing Machine in the sense discussed in class. Describe your TMs using a modular diagrammatic representation as done in class. (*10*)

   (a) $square : \mathbb{N} \to \mathbb{N}$ where $square(n) = n^2$.

   (b) (integer division) $div : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, where $div(m, n)$ is the largest integer less than or equal to m/n if $n > 0$, and 0 otherwise.