Automata Theory and Computability

Assignment 5 (Turing Machines and Decidability)

Total 55 marks. Due on Mon 2nd December 2019.

1. Is the following question decidable? Given a Turing machine M and a state q of M, does M ever enter state q on *some* input? (*5*)

2. An *enumeration* machine N is a two-tape Turing machine with the following distinctions:

   - The machine is not given any input; both its tapes are blank to begin with.
   - The first tape is a write-only tape, on which the machine can only write symbols of Σ. The second tape is a usual two-way read/write tape on which it can write any element of Γ.
   - The machine has no accept/reject state, but instead it has a special enumeration state e by which it signals that it has written something interesting on its first tape. Thus the contents of the first tape are said to have been "enumerated" whenever the machine enters the state e. After entering e, the contents of the first tape are "automatically" erased and the first tape head is rewound to the left end of the tape. The machine then resumes working from there.
   - The language $L(N)$ is defined to be the set of strings in Σ^* enumerated by N.

   (a) Prove that enumeration machines and Turing machines are equal in computation power: i.e. the class of languages they define is precisely the r.e. languages. (*5*)

   (b) Prove that an r.e. language is recursive iff there is an enumeration machine that enumerates it in *increasing* order. (*5*)

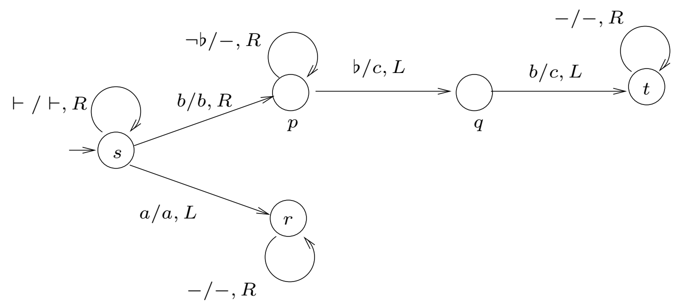
3. We say that a language L is *complete* for a class $\mathcal{C}$ of languages, if L belongs to $\mathcal{C}$, and whenever M is a language in $\mathcal{C}$ we have that $M \leq L$ (i.e. M reduces to L by the computable-map definition given in class). Argue that *HP* is complete for the class of recursively enumerable languages. (*5*)

4. Show that neither the language

   $$\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$$

   nor its complement is r.e. (*10*)

5. Consider the TM M below, with input alphabet $\{a, b\}$.



(a) Give any string in $\mathit{Valcomp}_{M,baabb}$. (5)

(b) Recall the notion of matching *triples* of symbols used in class. Give the entire set of matching triples for M. (5)

(c) Justify the claim that for two valid configurations c_1 and c_2 of M, which are of the same length, we have: $c_1 \stackrel{1}{\Rightarrow} c_2$ iff for each position in c_1, the triple of symbols in c_1 and the corresponding triple in c_2 match. (5)

6. (a) Consider a language L over the alphabet $A \cup \{\#\}$, whose strings are of the form $w_0\#w_1\#\cdots w_n\#$, with each $w_i \in A^*$ and all w_i's having the same length; further, L is given to be infinite. Argue that L cannot be a CFL. (5)

(b) Is it decidable whether the complement of a given CFL is a CFL? As usual, justify your answer. (5)