REGULARITY-PRESERVING RELATIONS

1. *Regularity-preserving relations - J.I Seiferas, R McNaughton*
2. *Automata Theory - Kozen*

Definition

For each binary relation r on the set $\mathbb{N}$ of nonnegative integers and each language L, define
$P(r, L) := \{x \mid \exists y \text{ s.t. } r(|x|, |y|) \text{ and } xy \in L\}$.

Definition

For each binary relation r on the set $\mathbb{N}$ of nonnegative integers and each language L, define
$P(r, L) := \{x \mid \exists y \text{ s.t. } r(|x|, |y|) \text{ and } xy \in L\}$.

Definition (Regularity preserving relation)

A relation r is regularity preserving if $P(r, L)$ is regular for every regular language L.

Definition (Ultimate periodicity)

A subset X of N is ultimately periodic if There exist $n_0 \geq 0$, $p \geq 1$ in N, such that for all $m \geq n_0$, $m \in X$ iff $m + p \in X$.

Definition (Ultimate periodicity)

A subset X of N is ultimately periodic if There exist $n_0 \geq 0$, $p \geq 1$ in N, such that for all $m \geq n_0$, $m \in X$ iff $m + p \in X$.

Definition (U.P. preserving relations)

A relation is U.P. preserving if for all ultimately periodic set A, the set

$$r^{-1}(A) := \{i | \ \exists j \in A \text{ s.t. } (i, j) \in r\}$$

is also U.P.

Characterisation of U.P. relations

Lemma

If L is regular, then $\{|x| \mid x \in L\}$ U.P.

Characterisation of U.P. relations

Lemma

If L is regular, then $\{|x| \mid x \in L\}$ U.P.

Lemma

If A is U.P, then $\{x \in \Sigma^* \mid |x| \in A\}$ is regular, for each finite alphabet Σ.

Characterisation of U.P. relations

Lemma

If L is regular, then $\{|x| \mid x \in L\}$ U.P.

Lemma

If A is U.P, then $\{x \in \Sigma^* \mid |x| \in A\}$ is regular, for each finite alphabet Σ.

Theorem

A relation is regularity preserving iff it is U.P.

U.P. degenerating relations

Definition

A binary relation r is U.P. degenerating if $r^{-1}(A)$ is finite/cofinite if A is finite/infinite respectively.

U.P. degenerating relations

Definition

A binary relation r is U.P. degenerating if $r^{-1}(A)$ is finite/cofinite if A is finite/infinite respectively.

Theorem

If r_1, r_2 are U.P. degenerating relations then $P(r_1, L) \setminus P(r_2, L)$ is finite for every regular language L.