Automata Theory and Computability

Assignment 3

(Total marks 40. Due on Mon 4th October 2021)

1. Describe the classes of the canonical Myhill-Nerode relation $\equiv_L$ for the language L over $\{a, b\}$, comprising strings with an odd number of a's and at most one b. Use it to construct the canonical DFA for L. *(5)*

2. Describe the equivalence classes of the canonical Myhill-Nerode relation $\equiv_L$ for the language $L = \{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. Use this to argue that L is not regular. *(10)*

3. Consider the monoid $M = (\{1, m, m'\}, \circ, 1)$ where $\circ$ is given by:

$\circ$	1	m	m'
1	1	m	m'
m	m	1	m'
m'	m'	m'	m'

   (a) Let A be the alphabet $\{a, b\}$. Describe a morphism $h : A^* \to M$, and a subset X of M, such that the language recognized by M, h, and X, is the language $\{b^k \mid k \text{ is even}\}$. *(5)*

   (b) Describe the syntactic monoid of this language. *(5)*

4. Give any language (other than the one given in class) whose synctactic congruence strictly refines its canonical MN relation. *(5)*

5. Show that recognizable languages over an alphabet A are closed under union. More precisely, given finite monoids M_1 and M_2 that accept languages L_1 and L_2 via morphisms and state-set pairs h_1, X_1 and h_2, X_2 respectively, show how to directly construct a monoid recognizing $L_1 \cup L_2$. *(5)*

6. Construct the transition monoid for the following DFA on three states $\{1, 2, 3\}$ over the alphabet $\{a, b, c\}$. *(5)*