Automata Theory and Computability

Assignment 4 (Context-Free Grammars)

(Total marks 65. Due on Mon 1st Nov 2021)

1. Give a language L over the alphabet $\{a, b\}$ which satisfies the property that neither L nor its complement contains an infinite regular language. *(10)*

2. Convert the following context-free grammar to Chomsky Normal Form: *(10)*

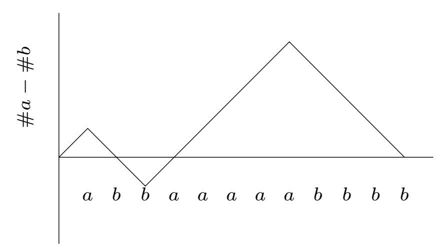
$$\begin{array}{rcl} S & \longrightarrow & AAA \mid B \\ A & \longrightarrow & aA \mid B \\ B & \longrightarrow & \epsilon \end{array}$$

3. Consider the grammar G below:

$$S \longrightarrow SS \mid aSb \mid bSa \mid \epsilon.$$

Prove that it generates the language $\{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. *(10)*

Hint: Consider the graph of a word x where you plot the value $\#_a(y) - \#_b(y)$ against prefixes y of x. Use induction as usual, and write your induction statement $P(n)$ clearlyl in each case.



4. Consider the language $L = \{a^n b^{n^2} \mid n \geq 0\}$. Use the Pumping Lemma for CFLs to show that L is not a CFL. *(10)*

5. Consider the CFG G below:

$$\begin{array}{rcl} S & \rightarrow & XC \mid AY \\ X & \rightarrow & aXb \mid ab \\ Y & \rightarrow & bYc \mid bc \\ A & \rightarrow & aA \mid a \\ C & \rightarrow & cC \mid c \end{array}$$

   (a) Describe the language accepted by G. *(5)*

   (b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is

i. Identify the basic pumps for G. *(5)*

ii. Identify the $\leq$-minimal parse trees. *(5)*

iii. Use these to obtain an expression for $\psi(L(G))$. *(5)*

(c) Use the semi-linear expression obtained above to give a regular expression that is letter-equivalent to $L(G)$. *(5)*