Assignment 2

E0 205 Mathematical logic and theorem proving

Weightage 5%. Due date: Wed 14 April 2021

1. Consider the propositional logic formula

$$\begin{array}{ll} c_1 & (p_{11} \vee p_{12}) \wedge \\ c_2 & (p_{21} \vee p_{22}) \wedge \\ c_3 & (\neg p_{11} \vee p_{22}) \wedge \\ c_4 & (\neg p_{12} \vee p_{21}) \wedge \\ c_5 & (\neg p_{22} \vee p_{12}). \end{array}$$

   (a) If the propositional symbol p_{ij} represents "pigeon i is in hole j", what does the formula state?

   (b) Use the DPLL algorithm to check satisfiability of the formula. Show the steps of BCP and finding conflict-clauses clearly.

2. Consider the propositional logic formula

$$\begin{array}{ll} c_1 & (p_{11} \vee p_{12}) \wedge \\ c_2 & (p_{21} \vee p_{22}) \wedge \\ c_3 & (p_{31} \vee p_{32}) \wedge \\ c_4 & (\neg p_{11} \vee p_{22}) \wedge \\ c_5 & (\neg p_{12} \vee p_{21}) \wedge \\ c_6 & (\neg p_{11} \vee p_{32}) \wedge \\ c_7 & (\neg p_{12} \vee p_{31}) \wedge \\ c_8 & (\neg p_{22} \vee p_{11}) \wedge \\ c_9 & (\neg p_{21} \vee p_{12}) \wedge \\ c_{10} & (\neg p_{21} \vee p_{32}) \wedge \\ c_{11} & (\neg p_{22} \vee p_{31}) \wedge \\ c_{12} & (\neg p_{11} \vee \neg p_{12}) \wedge \\ c_{13} & (\neg p_{21} \vee \neg p_{22}) \wedge \\ c_{14} & (\neg p_{31} \vee \neg p_{32}). \end{array}$$

   (a) If the propositional symbol p_{ij} represents "vertex i is coloured with colour j", what does the formula state?

   (b) Use the DPLL algorithm to check satisfiability of the formula. Show the steps of BCP and finding conflict-clauses clearly.

3. The DPLL algo's UNSAT conclusion for a CNF formula F can be viewed as a resolution-based refutation of F. We can picture such a resolution-based refutation as a binary resolution graph, with nodes labelled with clauses and and directed edges from clauses c_1 and c_2 to c_3 if c_3 is obtained by the resolution rule from c_1 and c_2. The original clauses in F are the roots of this tree, and the empty clause "[ ]" is a sink node.

   (a) Show this tree for the colouring formula F above.

   (b) An *UNSAT core* of such a proof is a subset of the original clauses, which is sufficient (using the same resolution graph) to derive the empty clause. Describe a way to compute a "minimal" UNSAT core from such a resolution graph.

   (c) Show the result of your proposed algorithm on the resolution graph of formula F.

4. Extend

$$Th = \{((P \supset Q) \vee ((\neg P) \wedge R)), \neg\neg Q, Q \wedge R, \neg S\}$$

   to a Hintikka theory and find a valuation to satisfy it.

5. (a) (Fitting 2nd ed., Table 2.3, page 30) gives rewrite rules and an algorithm to convert a propositional logic formula to conjunctive normal form. Prove that this algorithm terminates.

   (b) Suppose you are given a truth table evaluation of a formula A under all possible valuations, extending the basic truth tables given in (Fitting 2nd ed., Table 2.1, page 15). Extract a CNF from this truth table for A.

6. The Davis-Putnam splitting rule is given in (Fitting 2nd ed., Section 4.4, page 101). Prove that this rule preserves satisfiability. (The proof of Proposition 4.4.3 for the one-literal rule shows how the argument is to be structured.)