Assignment 3

E0 205 Mathematical logic and theorem proving

Weightage 12%. Due date for Q1,2,3: Mon 10 May 2021

1. Prove that an Equality Logic formula φ which is a conjunction of literals, is satisfiable iff its induced equality graph G_φ has no contradictory cycles. *(6)*

2. Consider the three approaches to checking satisfiability of Equality Logic formulas below:

   - DNF + Equality Graph: Convert the given E-formula φ into DNF, and check the satisfiability of each disjunct by checking for contradictory cycles in the induced equality graph.
   - DPLL(T): Convert the given E-formula φ to a PL formula $e(\varphi)$ by replacing equalities with new propositional variables. Check for satisfiability using a PL solver. If UNSAT, return UNSAT. If SAT, see if the assignment corresponds to a satisfiable equality assignment (for example using the induced equality graph). If it does, return SAT; else add a clause to $e(\varphi)$ corresponding to the conflicting equality assignments, and repeat.
   - Bryant's conversion to PL: Convert the given E formula φ to a PL formula φ' by replacing equalities with new propositional variables, making the induced non-polar equality graph chordal, and adding the "no contradictory triangle" clauses. Check for sat using a PL sat solver.

   Apply each of these approaches on each of the formulas below: *(12)*

   (a) $x_1 = x_2 \wedge x_1 = x_3 \wedge (x_1 \neq x_2 \vee f_1 = f_2) \wedge (x_2 \neq x_3 \vee f_2 = f_3) \wedge (f_1 \neq f_3)$

   (b) $x_1 = x_2 \wedge (x_2 = x_3 \vee f_2 = f_3) \wedge (f_1 \neq f_3) \wedge (x_1 = x_3 \Rightarrow f_1 = f_3) \wedge (x_2 = x_3 \Rightarrow f_2 = f_3)$

3. Consider the following approaches to checking satisfiability of EUF formulas:

   - DNF + Congruence Closure: Convert the given EUF-formula φ into DNF, and check the satisfiability of each disjunct using Shostak's Congruence Closure algorithm. Return SAT if any one disjunct is SAT, else return UNSAT.

- DPLL(T): Convert the given EUF-formula φ to a PL formula $e(\varphi)$ by replacing equalities with new propositional variables. Check for satisfiability using the native Z3 PL solver. If UNSAT, return UNSAT. If SAT, see if the assignment corresponds to a valid EUF assignment using Congruence Closure. If it does, return SAT; else add a clause to $e(\varphi)$ corresponding to the conflicting assignment, and repeat.
- Ackermann's Reduction: Convert the given EUF formula φ to an Equality Logic formula φ' using Ackermann's reduction. Apply any of the techniques for Equality Logic to check satisfiability of φ'.

Use each of the above approaches to check satisfiability of the following EUF formulas: *(12)*

(a) $x = F(F(F(F(F(x))))) \wedge x = F(F(F(x))) \wedge x \neq F(x)$

(b) $(G(F(x)) = x \wedge F(G(F(x))) = x \wedge F(G(x)) \neq x) \vee (F(G(x)) = x \wedge F(F(G(x))) = x \wedge F(x) \neq G(x))$

4. (Implementing sat approaches for Equality Logic) *(30)*

Implement the approaches in Q2 above for checking the satisfiability of Equality Logic formulas, using Z3/Python. Evaluate the time taken by the three approaches on 10 different input E-formulas, and say which one performs better, if any.